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A Holomorphic embedding approach for finding the Type-1 power-flow solutions

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ABSTRACT

A holomorphic embedding method (HEM)-based algorithm for finding Type-1 power-flow solutions is introduced whose complexity is the same as that of the HEM power-flow algorithm for calculating the high voltage (operable) power-flow solution. The algorithm is tailored to finding Type-1 solutions by using a modified embedded system and a numerical mapping from a set of integer-based boundary conditions to a floating-point-number-based reference state. The modified system can also be viewed as a homotopy whose initial point (no-load reference state) is consistent with the Type-1 solution homotopy path of the modified system, with an embedding parameter that functions simultaneously as the homotopy parameter. By using analytic continuation, starting from the initial point/reference state, the solution obtained for the modified system matches the one obtained from the original system model at the load of interest. Numerical results for three-/five-/seven-/14- and 118-bus systems are presented to demonstrate the numerical robustness.

1. Introduction

There exist multiple power-flow solutions (PFS's) for a power system characterized by the traditional complex nonlinear power-balance-equations (PBE's), though the exact number remains unknown. Efforts have been made to find all/multiple PFS's using various approaches [1–4]. Because of the number of possible solutions, finding all solutions, with or without imposing VAr limits, is a computationally formidable task. In this work, we limit ourselves to finding the solutions of most interest from a power system voltage stability assessment point of view, namely the Type-1 PFS's [5–7]. The power system voltage stability problem, exacerbated by the rise of loading levels without concomitant transmission expansion, has been determined to be responsible for several major blackouts across different countries. Various studies have been completed to analyze the voltage instability phenomenon and create metrics which measure voltage stability margin, among these metrics is the distance between high-voltage (HV) (or operable solution) and the Type-1 PFS's [8–11]. Much research effort has been expended in the development of algorithms and appropriate numerical methods for finding all Type-1 PFS's: In [8], an algorithm has been proposed for finding some Type-1 PFS's by applying Newton's method to the algebraic equations that characterize the solution points. The fundamental idea of the algorithm is to 'guess' at an initial estimate close to a Type-1 PFS, e.g., by setting the initial estimate for one of the bus voltages close to 0.0 instead of 1.0 (flat start), so that the iterative process will converge to a Type-1 solution. By selecting the estimate of each bus voltage, one-at-a-time, to be close to zero, the hope is to find all Type-1's. This algorithm suffers from the well-known shortcomings of Newton's method and will not necessarily converge to the desired solution even if the initial estimate is close to the solution [9,10]. A more reliable algorithm developed from the continuation power flow (CPF) method was proposed and numerically tested in [11]. The CPF-based method traces the PV curve (for a load bus) or the Pδ curve (for a generator bus) for each bus in the system, by varying the load/generation at only one bus at a time. By starting from a HV solution, where all eigenvalues of the power-flow (PF) Jacobian matrix have all negative real parts, the traced curve reaches and passes the saddle nose bifurcation node point (SNBP), with the sign of the real part of only one eigenvalue changing from negative to positive, resulting in a Type-1 PFS. While the theory in [11] is rigorous, the CPF-based numerical method is unsuccessful at finding all the Type-1 solutions for systems with both non-radial and weakly connected regions that have strong voltage support [12].

For systems with dynamic models included, the solutions of most interest in transient stability assessment through energy function methods are the Type-1 unstable equilibrium points (UEP's) ([13–18]). When classical machine models are included and the branch resistances are ignored, Type-1 UEP's problem become identical to Type-1 PFS's

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problem (with no additional state space variables [19]) and the methods used to find Type-1 UEP are intimately related to the methods proposed here (as will be shown) and inform research into the algorithms for finding the Type-1 PFS's. It is well known that the collection of Type-1 UEP's forms the energy function topographical boundary that, once violated by the system state, leads to instability. The set of Type-1 UEP's contains the closest UEP, which is of special interest in transient stability margin assessment. The closest UEP is the Type-1 UEP, whose energy function has the lowest value [13], excluding of course the energy level of the reference point, i.e., the stable equilibrium point (SEP). The authors in [19] propose a method for finding the closest UEP by redefining the problem statement such that a search for a Type-1 UEP is replaced by a search for the SEP of the reformed system. With the UEP of interest looking more like the SEP of the reformed system, the hypothesis was that any iterative method, which is reliable for SEP calculations, could be used by starting from a reasonable initial estimate. However, it was reported in [20] that only two Type-1 UEP's could be found by the method proposed in [19] for a system which is known to have more; hence this method offers no guarantee of finding all the Type-1 UEP's or of finding the closest UEP. The authors in [20] have developed a homotopy-based method for finding all the Type-1 UEP's for a power system, and then identifying the closest UEP. It was proven that if the homotopy curve passes the 'turning point' (similar to the SNBP but applied to the homotopy path) only once, the solution obtained will be a Type-1 UEP (This approach has similarities to the proposed CPF-based method in [11].) While the method proposed in [20] is more reliable for finding the closest UEP, it is computationally expensive to trace all the homotopy curves, computing converged solution after converged solution along each homotopy path. Additionally, it is possible to have multiple revisits of the same Type-1 solution as reported in [4], thus reducing the efficiency of the algorithm further.

The method proposed in this paper uses several of these principles to achieve a method for finding up to N Type-1 solutions (in an $(N+1)$-bus system), with a theoretical guarantee of convergence provided certain mild conditions are obeyed. The approach achieves its theoretical convergence guarantee by using the same technique used by HELM [21] for finding the HV solution of the PF problem, but uses reference states (RS's), corresponding to Type-1 solution branches (rather than the Type-0 solution branch which produces an HV solution), so that the Type-1 solution associated with each branch is found. The RS for the Type-1 branch is found for a modified system by using a numerical mapping from a set of integer-based boundary conditions to a floating-point-number-based RS. The modified system is constructed as a homotopy whose initial point (no-load RS) is consistent with the Type-1 solution branch of the modified system and then analytic continuation is used as both the system and load parameters are simultaneously modified by the same embedded/homotopy parameter so that the solution point matches the system model at the load of interest. A word about VAr limits and bus-type switching: often times the Type-1 PFS's are associated with large generator reactive power outputs, exceeding the VAr limits and resulting in bus type switching (from PV bus to PQ bus) [2]. To incorporate bus-type switching into the proposed work one would use an iterative approach: finding a possible Type-1 PFS, imposing VAr limits as indicated, and re-solving the modified system for another candidate Type-1 PFS, and so on. The effect of bus-type switching is to reduce the voltage stability margin, which affects all solutions [2]. The authors realize that this is a complex theoretical and numerical problem, deserving of an in-depth treatment which is regrettably beyond the scope of the present work.

This paper is organized as follows: In Section 2, the approach used is presented to help the reader visualize the concept and some comments about convergence issues are discussed. Section 3 introduces the HEM-based equations used to solve for the low-voltage/large-angle (LV/LA) PFS's that correspond to Type-1 PFS's. In Section 6, the method for finding the no-load RS's is introduced. In Section 5, the equations for

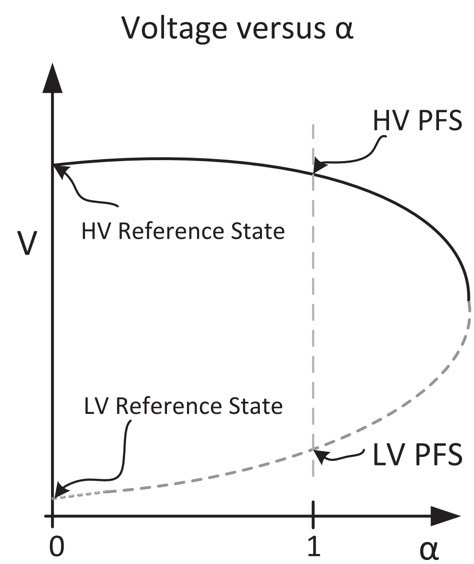

Fig. 1. Analytic continuation on a conventional PV curve.

finding the full-load solution are developed. In Section 6, several theorems establish the connection between the integer-based boundary conditions and the Type-1 solutions. In Section 7, numerical test results for different sample systems are given. The conclusions are presented in Section 8.

2. The concept

The approach is conceptually straightforward and based upon the HEM for finding the HV PFS [21,22]. Consider a two-bus system, with a slack bus and a PQ bus. The HV (black, solid) and LV (gray, short-dashes) of Fig. 1 represent the PV curves for the Type-0 and Type-1 PFS's, respectively. For such a two-bus system, where α represents the load scaling parameter, only Type-0 (HV) and Type-1 (LV) PFS's exist.

HEM is used for finding the HV solution by first calculating the HV no-load RS and then determining the Maclaurin series coefficients for the HV solution, provided a solution exists for the load profile given. Through the use of Padé approximants, which insure both that the series converges and convergences is accelerated [23], bus voltages are calculated. This is the approach used by the HELM commercial-grade PF software [21]. Convergence is guaranteed provided the PBE formulation is selected correctly and some mild continuity conditions are obeyed, i.e., the PBE's are continuous with no bifurcation points on the interval [0,1].

In this paper we show that we can use the same techniques to find the PFS corresponding to the low-voltage (LV) curve (Type-1 PFS) in Fig. 1: Rather than starting from the HV RS, the LV no-load RS is calculated and HEM is used to obtain the Maclaurin series coefficients of the LV curve while Padé approximants are used to deal with divergent and slowly convergent series. While the HV solution has (arguably) one RS for an $(N + 1)$-bus system, the LV solutions have at least 2^{N}-1 unique RS's, many of which introduce voltage zeros, which lead to undefined functions. In this work we show how to handle these voltage zeros using dual variables and how to identify RS's corresponding to N Type-1 PFS from the 2^{N}-1 possibilities.

<u>A word about the HEM convergence guarantees:</u> The HEM convergence guarantee, as it applied to the PFS is contingent on two aspects of the problem. First, unlike a Newton formulation in which a solution need only exist at the load of interest, HEM requires that solutions exist continuously along the analytic continuation path from the no-load RS to the load of interest and that bifurcations do not take place along this path. (More precisely, $\alpha = 1$ should not be a branching point, and it should not be contained in Stahl's cut-set (a specific case of the