Electrical Power and Energy Systems 102 (2018) 296–301

Contents lists available at ScienceDirect

Electrical Power and Energy Systems

journal homepage: www.elsevier.com/locate/ijepes

Thevenin's theorem approach to mutually coupled elements

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ARTICLE INFO

Keywords:
Thevenin's theorem
Thevenin's voltage and impedance
Mutual coupling
Bus admittance and impedance matrices

ABSTRACT

The aim of this paper is to apply Thevenin's theorem to mutually coupled elements. Mutual coupling between elements exist due to proximity of elements in electric circuits. The determination of the current in a new added element to an existing circuit is usually done through Thevenin's theorem when the new element is not mutually coupled with any other element in the circuit. However, when the element is mutually coupled with other elements in the circuit, then application of Thevenin's theorem is not straightforward. Thevenin's voltages and impedances have to be modified to include the effect of mutual coupling between the new element and the existing elements of the circuit. A systematic approach to modify Thevenin's voltages and impedances is presented in this paper. Two examples are presented to demonstrate the procedure. The first example considers only single coupling while the second considers multiple coupling. In single coupling, the new element is coupled only with a single element of the circuit. In multiple coupling, the element is coupled with several elements of the circuit.

1. Introduction

Many electrical circuit elements have mutual coupling between them due to proximity of elements. Such coupling is due to electric field proximity as in capacitors, or magnetic field proximity as in inductors. Formation and modification of bus admittance and impedance matrices of circuits containing mutually coupled elements has been treated thoroughly [1–4]. Mutual coupling between elements of antenna arrays have been investigated by several researchers [5–12]. The effects of mutual coupling between transmission lines in power systems have been investigated in [13–19]. Thevenin's theorem has been applied widely in many fields of electrical engineering due to its simplicity. The application of Thevenin's theorem to determine the current in a new added element to a circuit is straightforward when the added element is not mutually coupled with other elements of the circuit. However, when the added element is mutually coupled with other elements of the circuit, then Thevenin's impedances and voltages have to be modified to account for the mutual effect. So many applications of electrical engineering, ranging from electronics and communications to power systems, may face problems with mutually coupled elements and hence need a systematic approach to applying Thevenin's theorem to such problems. In this paper, a systematic approach to applying Thevenin's theorem to mutually coupled elements is presented. Thevenin's voltage between the nodes of the new added element is modified by a factor depending on the mutual coupling and Thevenin's voltages of the mutually coupled element nodes. Thevenin's impedance is modified by a factor depending on mutual coupling and Thevenin's impedances between the mutually coupled element nodes as well as between these nodes and the new added element nodes.

2. Mutual coupling with a single element

An external element of self-impedance z_{sf} is to be connected between nodes p and q. The element is mutually coupled with an existing element connected between nodes a and b that has self-impedance z_{sab} as shown in Fig. 1. The mutual impedance between both elements is z_m.

If the element is connected between the existing node p and a new node f as shown in Fig. 2, then the voltage and current relationship of both elements are

$$V_{ab} = z_{sab} I_{ab} + z_m I_f \tag{1}$$

$$V_{pf} = z_m I_{ab} + z_{sf} I_f \tag{2}$$

In matrix form

$$\begin{bmatrix} V_{ab} \\ V_{pf} \end{bmatrix} = \begin{bmatrix} z_{sab} & z_m \\ z_m & z_{sf} \end{bmatrix} \cdot \begin{bmatrix} I_{ab} \\ I_f \end{bmatrix} \tag{3}$$

$$\begin{bmatrix} I_{ab} \\ I_f \end{bmatrix} = \begin{bmatrix} z_{sab} & z_m \\ z_m & z_{sf} \end{bmatrix}^{-1} \cdot \begin{bmatrix} V_{ab} \\ V_{pf} \end{bmatrix} = \begin{bmatrix} y_{sab} & y_m \\ y_m & y_{sf} \end{bmatrix} \cdot \begin{bmatrix} V_{ab} \\ V_{pf} \end{bmatrix} \tag{4}$$

This yields

$$I_f = y_{sf} V_{pf} + y_m V_{ab} \tag{5}$$

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https://doi.org/10.1016/j.ijepes.2018.04.033
Received 30 November 2017; Received in revised form 22 March 2018; Accepted 25 April 2018
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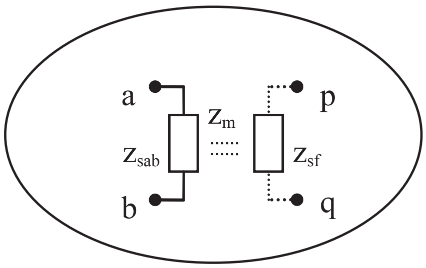

Fig. 1. Mutual coupling between elements.

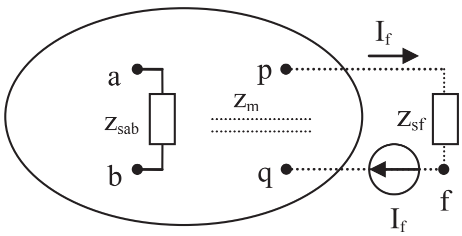

Fig. 2. Voltage and current relationship between coupled elements.

$$V_{pf} = y_{sf}^{-1}(I_f - y_m V_{ab}) \tag{6}$$

With $I_f = 0$, we have

$$V_{pf} = -y_{sf}^{-1} y_m V_{ab} \tag{7}$$

The new node voltage is

$$V_f = V_p - V_{pf} = V_p + y_{sf}^{-1} y_m V_{ab} \tag{8}$$

If the circuit has node (bus) impedance matrix, Z_{bus} of order n, and injected currents to the nodes are I, then we have

$$V = Z_{bus}.I \tag{9}$$

The voltage of node p is

$$V_p = \sum_{i=1}^{n} z_{pi} I_i \tag{10}$$

where z_{pi} are elements of the bus impedance matrix corresponding to node p and I_i is injected current at node i. The voltage across the element between nodes a and b is

$$V_{ab} = V_a - V_b = \sum_{i=1}^{n} (z_{ai} - z_{bi}) I_i \tag{11}$$

The new node voltage from (8) is

$$V_f = \sum_{i=1}^{n} z_{fi} I_i \tag{12}$$

where

$$z_{fi} = z_{pi} + y_{sf}^{-1} y_m (z_{ai} - z_{bi}) \tag{13}$$

are elements corresponding to the new node f row in the bus impedance matrix. The elements of the new column corresponding to node f are equal to those of its row (z_{fi}) due to the symmetry of the bus impedance matrix. With $I_f \neq 0$

$$V_p = \sum_{i=1}^{n} z_{pi} I_i + (z_{pp} + y_{sf}^{-1} y_m (z_{ap} - z_{bp}))\cdot(-I_f) \tag{14}$$

and from (6), V_f is

$$V_f = V_p - V_{pf} = V_p + y_{sf}^{-1} y_m V_{ab} + y_{sf}^{-1}(-I_f) \tag{15}$$

From the bus impedance matrix, V_f should be

$$V_f = \sum_{i=1}^{n} (z_{pi} + y_{sf}^{-1} y_m (z_{ai} - z_{bi})) I_i + z_{ff}(-I_f) \tag{16}$$

Note that V_a and V_b with $I_f \neq 0$ are

$$V_a = \sum_{i=1}^{n} z_{ai} I_i + z_{fa}\cdot(-I_f) \tag{17}$$

$$V_b = \sum_{i=1}^{n} z_{bi} I_i + z_{fb}\cdot(-I_f) \tag{18}$$

where z_{fa} and z_{fb} from (13) are

$$z_{fa} = z_{pa} + y_{sf}^{-1} y_m (z_{aa} - z_{ba}) \tag{19}$$

and

$$z_{fb} = z_{pb} + y_{sf}^{-1} y_m (z_{ab} - z_{bb}) \tag{20}$$

Eqs. (15) and (16) yield

$$z_{ff} = z_{pp} + y_{sf}^{-1} y_m (z_{ap} - z_{bp}) + y_{sf}^{-1} y_m (z_{fa} - z_{fb}) + y_{sf}^{-1} \tag{21}$$

or

$$z_{ff} = z_{fp} + y_{sf}^{-1}(1 + y_m (z_{fa} - z_{fb})) \tag{22}$$

Injecting current $-I_f$ to node f and I_f to node q and adjusting until $V_f = V_q$, yields

$$V_f = \sum_{i=1}^{n} z_{fi} I_i + z_{ff}(-I_f) + z_{fq} I_f \tag{23}$$

and

$$V_q = \sum_{i=1}^{n} z_{qi} I_i + z_{fq}(-I_f) + z_{qq} I_f \tag{24}$$

Equating V_f to V_q yields

$$\sum_{i=1}^{n} (z_{fi} - z_{qi}) I_i = (z_{ff} + z_{qq} - 2z_{fq}) I_f \tag{25}$$

Note that

$$z_{fi} - z_{qi} = (z_{pi} - z_{qi}) + y_{sf}^{-1} y_m (z_{ai} - z_{bi}) \tag{26}$$

and

$$E_{th(k)} = \sum_{i=1}^{n} z_{ki} I_i \tag{27}$$

is Thevenin's voltage of node k (prior to connecting the new element). Using (26) and (27) yields

$$\sum_{i=1}^{n} (z_{fi} - z_{qi}) I_i = E_{th(pq)} + \frac{y_m}{y_{sf}}.\, E_{th(ab)} \tag{28}$$

Note that z_{ff} from (21) and z_{fa} and z_{fb} from (19) and (20) yield

$$z_{ff} = z_{pp} + 2\frac{y_m}{y_{sf}}(z_{ap} - z_{bp}) + \left(\frac{y_m}{y_{sf}}\right)^2 (z_{aa} + z_{bb} - 2z_{ab}) + \frac{1}{y_{sf}} \tag{29}$$

and z_{fq} from (13) is

$$z_{fq} = z_{pq} + y_{sf}^{-1} y_m (z_{aq} - z_{bq}) \tag{30}$$

This yields

$$z_{ff} + z_{qq} - 2z_{fq} = \frac{1}{y_{sf}} + 2\frac{y_m}{y_{sf}}((z_{ap} - z_{aq}) - (z_{bp} - z_{bq})) + (z_{pp} + z_{qq} - 2z_{pq}) + \left(\frac{y_m}{y_{sf}}\right)^2 (z_{aa} + z_{bb} - 2z_{ab}) \tag{31}$$

Note that prior to connecting the new element,

$$z_{th(pq)} = z_{pp} + z_{qq} - 2z_{pq} \tag{32}$$