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Linear method for steady-state analysis of radial distribution systems

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ABSTRACT

Linear methods for steady-state analysis of distribution systems are getting more and more important due to the spreading of distributed energy resources, such as distributed generation, storage systems, active demand. This paper proposes a new linear method based on a Jacobian approach for radial distribution network with lateral derivations and distributed energy resources. The set of the linear equations modeling the distribution system is firstly presented and, then, solved in a closed form. It includes the full π-model for lines, ZIP model for uncontrolled loads, both P-Q and P-V control for distributed energy resources. The adoption of a peculiar set of modeling variables and the radial topology of the network allows to obtain high accuracy and low computational times. The effectiveness of the method is tested on both a 24-nodes and a 237-nodes network. The method is firstly applied to sensitivity analysis and compared with other linearized methods in terms of accuracy and computational efficiency; then, it is applied to the power flow analysis and compared with the classical non linear load flow.

1. Introduction

Linear methods for analyzing steady-state operation of distribution systems are gaining more and more importance in the planning and operation activities, due to the wide and rapid spread of distributed energy resources (DERs) (i.e. distributed generation, storage systems, active demand). Typical applications of linear methods are power flow analysis [1–3], power system optimization studies [4,5], power losses estimation [6], and sensitivity analysis for hosting capacity evaluation [7], for pricing and placement of DERs and control devices [8,9], for Volt/VAr control [10,11].

A widely-used approach in linear methods is to evaluate a given initial operating condition of the distribution system and, then, the sensitivity coefficients that linearly relate the variations of network electrical variables to parameter changes (i.e. powers injected by distributed generators, power exchanges by storage systems and by voltage control devices). In the present paper, this approach is adopted and attention is focused only on the impact of active and reactive powers injected/absorbed by DERs on the electrical variables of the network.

The initial operating condition is generally obtained by solving a single load-flow problem in a base-case [12]. On the other hand, several methods have been proposed to evaluate sensitivity coefficients, which can be classified into three main categories: perturb and observe methods, circuit theory methods and Jacobian-based methods.

Perturb and observe methods evaluate the sensitivity coefficients as numerical derivatives, that is the ratio between the finite variation of an electrical variable (observation) caused by an assigned DER power variation (perturbation). The variations are evaluated by either simulation [9] or actual measurements [13,14] or load-flow calculation [15,16]. Accuracy of these methods are strictly dependent on the evaluation technique (f.i. high for load-flow and low for measurements), whereas the computational efficiency is quite low if many DERs are to be considered.

The second category includes the circuit theory methods which derive the sensitivity coefficients from linear circuit equations, such as the network impedance matrix [10,15,17,18], the line voltage drop expression [11], the two-port network equations [19], the adjoint network [20]. These methods present a trade-off between accuracy of the results and computational efficiency, because the latter one can be improved only by introducing model approximations and, consequently, reducing accuracy.

The third category includes the Jacobian-based methods which, in their classical formulation, obtain the sensitivity coefficients from the inverse of the Jacobian matrix, derived from the load-flow solution [4,8]. Since these methods use the analytical derivatives, they are the most accurate but, on the other hand, present two main drawbacks: *i.* the Jacobian matrix is available from the load-flow solution obtained by the Newton-Raphson technique, which presents well-known convergence problems in distribution systems; *ii.* the computational efficiency significantly decreases with the increase of the number of the

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Nomenclature

P_{MV},Q_{MV} powers injected by the MV voltage source
P_{LV},Q_{LV} powers in-flowing LV busbars
P_n,Q_n powers out-flowing the node n
P_n^{in},Q_n^{in} powers in-flowing series parameters of the π-model of the line n
P_n^{out},Q_n^{out} powers out-flowing series parameters of the π-model of the line n
P_n^{line},Q_n^{line} powers out-flowing the line n
P_n^{load},Q_n^{load} power consumptions at the node n
P_n^{der},Q_n^{der} powers injected by DERs at the node n
P_n^{lat},Q_n^{lat} powers flowing into the lateral derived from the node n
V_{MV}^2 square voltage at the MV busbars
V_{LV}^2 square voltage at the LV busbars
V_n^2 square voltage amplitude at the node n

nodes in the network, due to the Jacobian matrix inversion. To overcome the first drawback, in [16] the analytical derivatives are derived from the Distflow equations [21] exploiting the radial topology of distribution networks. However, the method in [16] does not overcome the second drawback. In fact, for each DER power variation, the related sensitivity coefficients are evaluated by solving a large set of linear equations.

In the present paper, extending and improving the approach presented in [22], a novel Jacobian-based method for the steady-state analysis of distribution systems is proposed by exploiting the radial network configuration, typically adopted in distribution system operation. Starting from an initial operating point, the proposed method directly provides the closed-form analytical expressions of the sensitivity coefficients, which linearly relate the variations of the active and reactive power flows and of the square nodal voltage amplitudes to the powers injected/absorbed by each DER connected to the network.

The main contributions of the paper are: (i) the model of the basic element, namely the line-node component (LNC), is extended to account for the full π-model of lines, for voltage-dependent load models, for different types of DER controls, and for lateral derivations; (ii) the set of the equations representing the linear model of the whole distribution system is provided, in which the adoption of a new set of variables for the analytical derivatives assures an improvement of the accuracy of the results with respect to other Jacobian-based methods; (iii) the closed form solution of the low-voltage (LV) distribution system is derived increasing the computational efficiency, especially for large distribution networks; (iv) the algorithm implementing the closed-form solution is outlined. The proposed model is developed with reference to balanced operating conditions, but its extension to unbalanced distribution systems is viable and will be presented in future studies.

The paper is organized as follows. In Section 2, the modeling equations of the supplying system as well as of the LV network are presented. In Section 3, the linearized model of the distribution system is firstly derived and, then, analytically solved thus obtaining its closed-form expression. Eventually, the method is firstly applied to sensitivity analysis and compared with other linearized methods in terms of accuracy and computational efficiency; then, it is applied to the power flow analysis and compared with the classical non linear load flow.

2. Distribution system modeling

Fig. 1 shows a typical LV distribution system with radial topology composed of a supplying system and the LV network. Reference is made to a LV distribution system but the analysis can be applied to medium-voltage (MV) distribution systems operating in radial configuration. Assumptions related to the modeling of the supplying system and of the LV network, which includes uncontrolled loads and DERs (i.e. photovoltaic generators, controllable loads, and storage systems), are described in the following.

2.1. Supplying system

The supplying system includes the MV distribution system and a MV/LV transformer. The former one is modeled by a voltage source imposing the no-load voltage at the MV busbar V_{MV}^2, in series with the

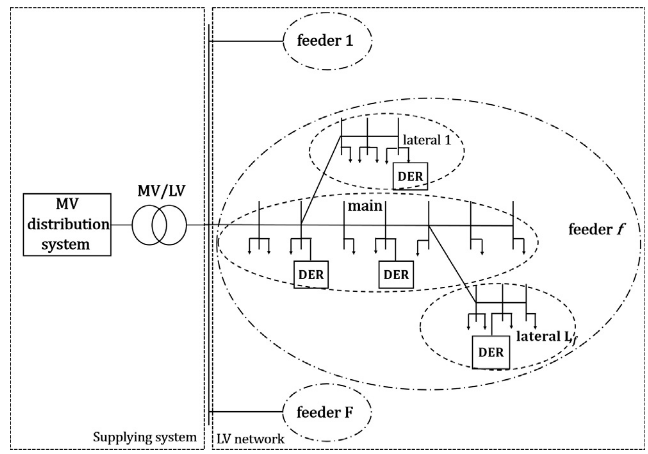

Fig. 1. Radial LV distribution system.

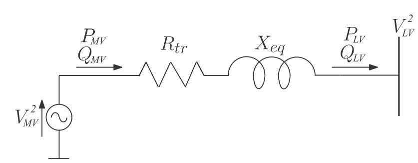

Fig. 2. Equivalent circuit of the supplying system.

short-circuit impedance X_{cc}. The transformer is modeled by its series parameters (i.e. resistance R_{tr} and leakage reactance X_{tr}). The equivalent circuit of the supplying system is shown in Fig. 2, being $X_{eq} = X_{cc} + X_{tr}$, and described by the Distflow equations [21]:

$$\begin{aligned} P_{MV} &= P_{LV} + R_{tr}\,(P_{LV}{}^2 + Q_{LV}^2)/V_{LV}^2 \\ Q_{MV} &= Q_{LV} + X_{eq}\,(P_{LV}{}^2 + Q_{LV}^2)/V_{LV}^2 \\ V_{MV}^2 &= V_{LV}^2 + 2(R_{tr}P_{LV} + X_{eq}Q_{LV}) \\ &\quad + (R_{tr}{}^2 + X_{eq}^2)(P_{LV}{}^2 + Q_{LV}^2)/V_{LV}^2 \end{aligned} \tag{1}$$

2.2. LV network

The LV network includes several feeders, each one composed of a main and different laterals. Each main and lateral is represented by a series of line-node components (LNCs). The generic nth LNC is composed of (Fig. 3): *i.* the line n between nodes $n-1$ and n; *ii.* the node n.

The line is modeled by the π equivalent circuit with series parameters (resistance R_n and reactance X_n) and shunt parameters (conductance G_n and susceptance B_n). Applying Kirchhoff laws and Distflow equations to the circuit in Fig. 3 yields:

$$\begin{aligned} P_n^{out} &= P_n^{in} - R_n\,(P_n^{in\,2} + Q_n^{in\,2})/V_{n-1}^2 \\ Q_n^{out} &= Q_n^{in} - X_n\,(P_n^{in\,2} + Q_n^{in\,2})/V_{n-1}^2 \\ V_n^2 &= V_{n-1}^2 - 2(R_nP_n^{in} + X_nQ_n^{in}) \\ &\quad + (R_n{}^2 + X_n^2)(P_n^{in\,2} + Q_n^{in\,2})/V_{n-1}^2 \end{aligned} \tag{2}$$