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Formal analysis of continuous-time systems using Fourier transform

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ABSTRACT

To study the dynamical behavior of the engineering and physical systems, we often need to capture their continuous behavior, which is modeled using differential equations, and perform the frequency-domain analysis of these systems. Traditionally, Fourier transform methods are used to perform this frequency domain analysis using paper-and-pencil based analytical techniques or computer simulations. However, both of these methods are error prone and thus are not suitable for analyzing systems used in safety-critical domains, like medicine and transportation. In order to provide an accurate alternative, we propose to use higher-order-logic theorem proving to conduct the frequency domain analysis of these systems. For this purpose, the paper presents a higher-order-logic formalization of Fourier transform using the HOL-Light theorem prover. In particular, we use the higher-order-logic based formalizations of differential, integral, transcendental and topological theories of multivariable calculus to formally define Fourier transform and reason about the correctness of its classical properties, such as existence, linearity, time shifting, frequency shifting, modulation, time scaling, time reversal and differentiation in time domain, and its relationships with Fourier Cosine, Fourier Sine and Laplace transforms. We use our proposed formalization for the formal verification of the frequency response of a generic n-order linear system, an audio equalizer and a MEMs accelerometer, using the HOL-Light theorem prover.

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1. Introduction

Fourier Transform (Bracewell, 1978) is a transform method, which converts a time varying function to its corresponding ω-domain representation, where ω is its corresponding angular frequency (Beerends et al., 2003). This transformation allows replacing the differentiation and integration in time domain analysis to multiplication and division operators in the frequency domain, which can be easily manipulated. Moreover, the ω-domain representations of the differential equations can also be used for the frequency response analysis of the corresponding systems. Due to these distinguishing features, Fourier transform has been widely used for analyzing many continuous-time systems, such as signal (Papoulis, 1977; Gaydecki, 2004), and image (Dougherty, 2009) processing algorithms, analog circuits (Thomas et al., 2016), communication systems (Ziemer and Tranter, 2006; Du and Swamy, 2010), medical sciences (Bracewell, 1978; Dougherty, 2009), mechanical systems (Oppenheim et al., 1996) and optics (Gaskill, 1978; Stark, 2012).

The first step in the Fourier transform based analysis of a continuous-time system, is to model the dynamics of the system using a differential equation. This differential equation is then transformed to its equivalent ω-domain representation by using the Fourier transform. Next, the resulting ω-domain equation is simplified using various Fourier transform properties, such as existence, linearity, frequency shifting, modulation, time shifting, time scaling, time reversal and differentiation. The main objective of this simplification is to either solve the differential equation to obtain values for the variable ω or obtain the frequency response of the system corresponding to the given differential equation. The frequency response can in turn be used to analyze the dynamics of the system by studying the impact of different frequency components on the intended behavior of the given system. The information sought from this analysis plays a vital role in analyzing reliable and performance efficient engineering systems.

Traditionally, the analysis of continuous-time systems, using transform methods, has been done using the paper-and-pencil based analytical technique. However, due to the highly involved human manipulation, the analysis process is error prone, especially when dealing with larger systems, and hence an accurate analysis cannot be guaranteed. Moreover, this kind of manual manipulation does not guarantee that each and every assumption required in the mathematical analysis is written down with the analysis. Thus, some vital assumptions may not accompany the final result of the analysis and a system designed based on such a result may lead to bugs later on. For example, the Air France Flight 447 crashed in 2009, which resulted in 228 deaths, was attributed to the faulty warning system consisting of speed sensors. These sensors gave wrong/invalid reading about the speed of the airplane, which led to the crash. A more rigorous analysis of the warning system could have prevented this incident.

Computer-based methods including the numerical methods and the symbolic techniques, provide a more scalable option to analyze larger systems. Some of the computer tools involved in these analysis are MATLAB (MATLAB, 2017), Mathematica (Wolfram, 2015) and Maple (Maple, 2017). The numerical analysis involves the approximation of the continuous expressions or the continuous values of the variables due to the finite precision of computer arithmetic, which compromises the accuracy of the analysis. Moreover, it involves a finite number of iterations, depending on the computational resources and computer memory, to judge the values of unknown continuous parameters, which introduces further inaccuracies in the analysis as well. Similarly, the symbolic tools cannot assure absolute accuracy as they involve discretization of integral to summation while evaluating the improper integral in the definition of Fourier transform (Taqdees and Hasan, 2013). Moreover, they also contain some unverified symbolic algorithms in their core (Durán et al., 2013), which puts another question mark on the accuracy of the results. Given the widespread usage of the continuous-time systems in many safety-critical domains, such as medicine and transportation, we cannot rely on these above-mentioned analysis methods as the analysis errors could lead to disastrous consequences, including the loss of human lives.

Formal methods (Hasan and Tahar, 2015) are computer based mathematical techniques that involve the mathematical modeling of the given system and the formal verification of its intended behavior as a mathematically specified property, which is expressed in an appropriate logic. This verification of the properties of the underlying system is based on mathematical reasoning. Moreover, the