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Motion planning and control of a planar polygonal linkage ☆

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ABSTRACT

For a polygonal linkage, we produce a fast navigation algorithm on its configuration space. The basic idea is to approximate $M(L)$ by the vertex-edge graph of the cell decomposition of the configuration space discovered by the first author. The algorithm has three aspects: (1) the number of navigation steps does not exceed 15 (independent of the linkage), (2) each step is a disguised flex of a quadrilateral from one triangular configuration to another, which is a well understood type of flex, and (3) each step can be performed explicitly by adding some extra bars and obtaining a mechanism with one degree of freedom.

1. Introduction

In the paper we work with a polygonal linkage (equivalently, with a flexible polygon), that is, with a collection of rigid bars connected consecutively in a closed chain. We allow any number of edges and any lengths assignments, (under a necessary assumption of the triangle inequality, which guarantees the closing possibility). The flexible polygon lives in the plane and admits different shapes, with allowed self-intersections. Taken together, all the shapes form the moduli space of the linkage.

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In the paper, we produce a motion planning algorithm (equivalently, a navigation algorithm) which explicitly reconfigures one shape to another via some continuous motion. In the language of the moduli space this means that we present a path leading from one prescribed point to another. We not only indicate the path, but also present a way of forcing the linkage to follow the path.

Although the problem does not seem very complicated (since the more edges we have, the more degrees of freedom we have), the navigation is not an easy issue because of the (possible) topological complexity of the moduli space.

There exists (see Lenhart and Whitesides, 1995) an $O(n)$ algorithm, where each step is a *line-tracking* motion. That is, during each step, the entire polygon except for some pentagonal subchain is frozen, only the subchain flexes in such a way that one of its vertices moves along a straight line.

Our reconfiguring algorithm is based on a stratification of the moduli space into a cell complex, introduced in Panina (2017). More precisely, we treat the one-skeleton of the complex as an appropriate approximation of the moduli space. In other words, we have an embedded graph, and we mostly navigate along the graph. The navigation goes as follows: from a given configuration of the linkage, we first reach an appropriate vertex of the graph, then navigate along the graph until we are close to the target configuration, and next, we pass to the target configuration. There are three important aspects about the algorithm:

(1) The number of steps (i.e., the number of edges of the connecting path) never exceeds 15. That is, we have a finite time algorithm (rather than n or even $log\ n$ complexity).
(2) However, finding each of the 15 designated configurations requires a linear time complexity algorithm.
(3) Each of the steps (that is, going along an edge of the graph) is a disguised flex of a quadrilateral polygonal linkage, which is both simple and well-understood.
(4) Each of the steps can be performed explicitly by adding some extra bars and obtaining a mechanism with one degree of freedom, see Section 4.

The paper is organized as follows. Section 2 gives precise definitions and explains the cell structure on the configuration space. We also present introductory examples and give a formula for the number of vertices of vertex-edge graph of the complex Γ. Section 3 explains the navigation on the graph. We show that a vertex-to-vertex navigation requires at most 15 steps.

Our next goal is to control the prescribed motions. We work under assumption that we have a full control of convex configurations, that is, we know how to reconfigure one convex configuration to another. There are different approaches how to do this: by using Coulomb potential, as in Khimshiashvili et al. (2014), by mechanically controlling the angles, in the way described in Aichholzer et al. (2001), or in some other way, not to be discussed in the paper. However we stress that for navigating over edges of the graph, it suffices to control just quadrilaterals, which is a much easier task, and which is well understood in all respects.

Navigation from an arbitrary point of the moduli space to a vertex requires one more step: we need to connect the starting point to the graph. Two different ways of doing this are described in Section 4.

The results presented in this paper arose as a natural continuation of the research on Morse functions on moduli spaces of polygonal linkages started in Khimshiashvili (2012), Panina and Khimshiashvili (2012), and Khimshiashvili et al. (2014). Several approaches to navigation and control of mechanical linkages have previously been discussed, in particular, in Hausmann (2005), Hausmann and Rodriguez (2007), Khimshiashvili (2012) and Khimshiashvili et al. (2014).

2. Moduli spaces of planar polygonal linkages

We start by a short review of some results on polygonal linkages and their moduli spaces.

A *polygonal n-linkage* is a sequence of positive numbers $L = (l_1, \ldots, l_n)$. It should be interpreted as a collection of rigid bars of lengths l_i joined consecutively in a chain by revolving joints. In the literature, it is sometimes called *a closed chain*.