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Stochastic stability of discrete-time Markovian jump delay neural networks with impulses and incomplete information on transition probability

Yu Zhang *

Department of Mathematics, Tongji University, Shanghai, 200092, PR China

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ABSTRACT

The purpose of this letter is to investigate the stochastic stability for a class of discrete-time Markovian jump delay neural networks with impulses and incomplete information on transition probability. By using Lyapunov functionals, some new results are provided. The obtained results show that impulses can stochastically stabilize an unstable discrete-time Markovian jump delay neural network. The obtained results also show that the stability property of the impulse-free neural network can be retained even under certain destabilizing impulses. Two examples together with their simulations are also presented to show the effectiveness and the advantage of the obtained results.

1. Introduction

In recent years, neural networks have attracted the interest of many researchers due to the fact that they have numerous applications in different areas such as signal processing, robotics, associative memory, fault diagnosis and pattern recognition, see Ahmad and Stamova (2008), Allegretto, Papini, and Forti (2010), Chua and Yang (1988), Gopalsamy (2004), Ho, Liang, and Lam (2006), Huang, Ho, and Qu (2007), Kaslik and Sivasundaram (2011), Kovacic (1993), Li and Shen (2010), Li, Shi, and Sun (2011), Li, Wu, Feng, and Liao (2011), Liu, Wang, Liang, and Liu (2009), Lou and Cui (2007), Mohamad (2001), Sakthivel, Raja, and Anthoni (2011), Stamova and Ilarionov (2010), Syed Ali and Marudai (2011), Tino, Cernansky, and Benuskova (2004), Wang, Liu, Yu, and Liu (2006), Wu, Shi, Su, and Chu (2012), Xu, Chen, and Teo (2010), Zhang (2011, 2012), Zhang and Chen (2008), Zhu and Cao (2012), and the references therein. Meanwhile, considering the stochastic neural networks with Markovian switching is of great significance since this kind of neural networks is very appropriate to model the neural networks subject to sudden environmental changes, random variation in the network's structures, changes in the interconnections of neurons, etc. Recently, the analysis of Markovian jump neural networks has attracted a great deal of research interest, see, e.g., Huang et al. (2007), Kovacic (1993), Liu et al. (2009), Lou and Cui (2007), Tino et al. (2004), Sakthivel et al. (2011), Syed Ali and Marudai (2011), Wang et al. (2006), Wu et al. (2012), Zhu and Cao (2012). In a Markovian jump neural network, the transition probabilities in the jumping process determine the neural network's behavior to a large extent. As mentioned in Zhang and Boukas (2009), obtaining the ideal information on all transition probabilities is questionable or generally expensive. Thus it is necessary to further consider the more general Markovian jump neural networks with incomplete transition probabilities.

In the implementation of neural networks, time delays always exist in the signal transmission among the neurons. The existence of time delays may lead to oscillation or instability of the neural networks. This is very harmful for the applications of neural networks. At the same time, the states of electronic networks are often subject to instantaneous changes and experience abrupt changes at certain instants, which can be caused by frequency change, switching phenomenon, or by the effect of some sudden noise, i.e., impulsive effects are also likely to exist in the neural networks (see, e.g., Sakthivel et al., 2011, Zhu & Cao, 2012). The impulses will affect the dynamical behaviors of the neural networks. Therefore, impulse effect and time delays should also be taken into account when investigating the stability of neural networks.

Though some results have been obtained for Markovian jump delay neural networks with impulse effect recently, see, e.g. Sakthivel et al. (2011), Zhu and Cao (2012). It is worth mentioning that Sakthivel et al. (2011), Zhu and Cao (2012) pertain to the stability analysis for continuous-time Markovian jump delay neural networks with impulses. However, many phenomena are described by discrete-time systems and in the fields of engineering

* Tel.: +86 21 65983241 2117; fax: +86 21 65981985.
E-mail addresses: zhangyu2008@tongji.edu.cn, zyconcept@163.com.

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especially the numerical simulation, the models always are discrete-time systems (see, for instance, Costa & Fragoso, 1993, Kaslik & Sivasundaram, 2011, Li, Wu et al., 2011, Liu et al., 2009, Mohamad, 2001, Pan, Sun, & Zhao, 2008, Syed Ali & Marudai, 2011, Wu et al., 2012, Xu et al., 2010, Zhang, 2012, Zhang & Boukas, 2009, Zhang, Sun, & Feng, 2009). Therefore, discrete-time Markovian jump delay neural networks have attracted the interest of some researchers, see Liu et al. (2009), Syed Ali and Marudai (2011), Wu et al. (2012) and the references therein. But not much attention has been paid to discrete-time Markovian jump delay neural networks with impulses.

Therefore, in the present letter, we will consider the stochastic stability of discrete-time Markovian jump delay neural networks with impulses and incomplete information on transition probability. The Lyapunov functionals will be used to obtain some new results. The results we obtained show that an impulse-free neural network, which is not stochastically stable, can be stochastically stable under certain stabilizing impulses. We also obtain the stability criteria that show when the impulse-free neural network is stochastically stable, then the stability property can be retained even with destabilizing impulses if the time interval between the nearest two impulses is appropriately large. It should be mentioned that the condition that the impulsive time interval is larger than 2 or the time delay is not necessary in the main results.

This letter is organized as follows. In Section 2, we introduce some basic definitions and notations. In Section 3, we get some stochastic stability criteria for discrete-time Markovian jump delay neural networks with impulses and incomplete information on transition probability. Two examples together with their simulations are also presented to illustrate the effectiveness and the advantage of the proposed results. Finally, concluding remarks are given in Section 4.

2. Preliminaries

Throughout this letter, if not explicitly stated, matrices are assumed to have compatible dimensions. $\mathbb{N}$ denotes the set of nonnegative positive integer numbers, i.e., $\mathbb{N} = \{0, 1, 2, \ldots\}$. $\mathbb{Z}^+$ denotes the set of positive integer numbers, $\mathbb{R}$ denotes the set of real numbers. For matrix M, the notation $M > (\geq, <, \leq)0$ is used to denote a symmetric positive definite (positive semidefinite, negative, negative semidefinite) matrix. $\lambda_{\min}(\cdot), \lambda_{\max}(\cdot)$ denote the minimum and the maximum eigenvalue of the corresponding matrix, respectively. $\|\cdot\|$ refers to the Euclidean vector norm. $E\{\cdot\}$ stands for the mathematical expectation. I and 0 represent, respectively, the identity matrix and zero matrix.

Consider the following discrete-time Markovian jump delay neural networks with impulsive perturbations in a fixed complete probability space.

$$\begin{cases} x(k+1) = A(r(k))x(k) + B(r(k))f(x(k)) \\ \qquad +C(r(k))g(x(k-d)), \quad k \neq \tau_j; \\ x(\tau_j+1) = D(r(\tau_j))x(\tau_j) \\ \qquad +E(r(\tau_j))x(\tau_j - d), \quad j \in \mathbb{Z}^+; \\ x(i) = \varphi(i), \quad i \in J = \{-d, -d+1, \ldots, -1, 0\}, \end{cases} \tag{1}$$

where $k \in \mathbb{N}$, $x(k) = (x_1(k), x_2(k), \ldots, x_m(k))^T \in \mathbb{R}^m$, $\{r(k), k \in \mathbb{N}\}$ is a discrete-time homogeneous Markov chain with finite state space $\mathbb{S} = \{1, 2, \ldots, s\}$, $A(r(k))$, $B(r(k))$, $C(r(k))$, $D(r(k))$, $E(r(k))$ $(r(k) \in \mathbb{S})$ are known real constant matrices, they represent the interconnection matrices representing the weight coefficients of the neurons. d is a positive integer denoting the constant delay time of the state in the system, $f(x(k)) = [f_1(x_1(k)), f_2(x_2(k)), \ldots, f_m(x_m(k))]^T$ and $g(x(k-d)) = [g_1(x_1(k-d)), g_2(x_2(k-d)), \ldots, g_m(x_m(k-d))]^T$ are the neuron activation functions, $f(0) = g(0) = 0$, $\varphi : J \to \mathbb{R}^m$, $\tau_0 = 0$, $\{\tau_j\} \subseteq \mathbb{Z}^+$ for $j \in \mathbb{Z}^+$, $0 < \tau_1 < \tau_2 < \cdots < \tau_j < \cdots$, $\tau_j \to \infty$ for $j \to \infty$.

Denote the state transition matrix by $\Pi = (\pi_{ij})_{i,j\in\mathbb{S}}$, i.e., the transition probabilities of $\{r(k), k \in \mathbb{N}\}$ are given by

$$Pr\{r(k+1) = j | r(k) = i\} = \pi_{ij} \quad \text{for } i, j \in \mathbb{S},$$

with $\pi_{ij} \geq 0$ for $i, j \in \mathbb{S}$ and $\sum_{j=1}^{s} \pi_{ij} = 1$ for $i \in \mathbb{S}$. The transition probabilities are considered to be partially available, that is, some elements in matrix Π are unknown. We will use $\hat{\pi}_{ij}$ to denote the unknown element $\pi_{ij} \in \Pi$.

Let $|\varphi| = \max_{i\in J} \|\varphi(i)\|$.

For convenience, for $\forall i \in \mathbb{S}$, we denote

$$\mathbb{S}_k^{(i)} \triangleq \{j : \pi_{ij} \text{ is known}\}, \qquad \mathbb{S}_{uk}^{(i)} \triangleq \{j : \pi_{ij} \text{ is unknown}\},$$
$$\mathcal{P}_k^{(i)} \triangleq \sum_{j\in\mathbb{S}_k^{(i)}} \pi_{ij}.$$

We have the following assumptions:

(A1) the activation functions, $f_i(x_i(k))$, $i = 1, 2, \ldots, m$ are assumed to be bounded and there exist scalars $\underline{k}_i$, $\bar{k}_i$ such that for any $\alpha, \beta \in \mathbb{R}$, $\alpha \neq \beta$,

$$\underline{k}_i \leq \frac{f_i(\alpha) - f_i(\beta)}{\alpha - \beta} \leq \bar{k}_i.$$

(A2) the activation functions, $g_i(x_i(k-d))$, $i = 1, 2, \ldots, m$ are assumed to be bounded and there exist scalars $\underline{l}_i$, $\bar{l}_i$ such that for any $\alpha, \beta \in \mathbb{R}$, $\alpha \neq \beta$,

$$\underline{l}_i \leq \frac{g_i(\alpha) - g_i(\beta)}{\alpha - \beta} \leq \bar{l}_i.$$

Definition 1. The discrete-time Markovian jump delay neural network with impulsive perturbations (1) is said to be stochastically stable, if for every initial state $(\varphi, r(0))$, the following holds:

$$E\left\{\sum_{k=0}^{\infty} \|x(k; \varphi, r(0))\|^2 | \varphi, r(0)\right\} < \infty.$$

3. Main results

Now, we consider the discrete-time impulsive Markovian jump delay neural network (1) with incomplete information on transition probability, we have the following results.

Theorem 3.1. *Let $\mu \geq 1$, $0 < c < 1$ be given constants. If there exist symmetric positive definite matrices $P(i) > 0$ $(i = 1, 2, \ldots, s)$, $Q_j > 0$ $(j = 1, 2, \ldots, d)$, $H_l = \mathrm{diag}(h_{l1}, h_{l2}, \ldots, h_{lm}) > 0$ $(l = 1, 2)$, constant $\zeta \in \mathbb{Z}^+$, such that the following inequalities hold for all $i \in \mathbb{S}$,*

$$\Omega_i(j) = \begin{pmatrix} \Omega_{11} & 0 & \Omega_{13} & A^T(i)\Psi_{ij}C(i) \\ \star & -H_2L_2 - \mu Q_d & 0 & H_2L_1 \\ \star & \star & \Omega_{33} & B^T(i)\Psi_{ij}C(i) \\ \star & \star & \star & -H_2 + C^T(i)\Psi_{ij}C(i) \end{pmatrix} < 0,$$
$$\forall j \in \mathbb{S}_{uk}^{(i)}, \tag{2}$$

where $\Omega_{11} = A^T(i)\Psi_{ij}A(i) + Q_1 - \mu P(i) - H_1K_2$, $\Omega_{13} = A^T(i)\Psi_{ij}B(i) + H_1K_1$, $\Omega_{33} = B^T(i)\Psi_{ij}B(i) - H_1$, $\Psi_{ij} = \sum_{l\in\mathbb{S}_k^{(i)}} \pi_{il}P(l) + (1 - \mathcal{P}_k^{(i)})P(j)$, $K_1 = \mathrm{diag}(\frac{\bar{k}_1+\underline{k}_1}{2}, \frac{\bar{k}_2+\underline{k}_2}{2}, \ldots, \frac{\bar{k}_m+\underline{k}_m}{2})$, $K_2 = \mathrm{diag}(\bar{k}_1\underline{k}_1, \bar{k}_2\underline{k}_2, \ldots, \bar{k}_m\underline{k}_m)$, $L_1 = \mathrm{diag}(\frac{\bar{l}_1+\underline{l}_1}{2}, \frac{\bar{l}_2+\underline{l}_2}{2}, \ldots, \frac{\bar{l}_m+\underline{l}_m}{2})$, $L_2 = \mathrm{diag}(\bar{l}_1\underline{l}_1, \bar{l}_2\underline{l}_2, \ldots,$