



# A subspace predictive control method for partially unknown systems with parameter learning event-triggered law

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## ABSTRACT

This paper proposes a novel event-triggered subspace predictive control (SPC) method for a class of linear discrete-time partially unknown systems. Without the complete system parameter information, the design parameters of the event-triggered law are first derived via system data by the reinforcement learning method. The proposed event-triggered law depends on the defined input error and the state-dependent threshold. The receding horizon principle in the typical predictive control methods is substituted by the event-triggered law, which can ensure the stability of the predictive input with optimality. The proposed method can considerably reduce the data computation and transmission load of the conventional SPC methods. The simulation results illustrate the effect and the satisfactory performance of the proposed method.

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## 1. Introduction

The subspace predictive control (SPC) approaches developed by Favoreel and Woodley [1,2] circumvent the modeling step and directly seeks the predictors of future outputs from data as a new predictive control method. The SPC method designs a predictive controller directly from the measured input and output (I/O) data without the system model. The key step of SPC is to identify a future output predictor by subspace identification, which maps the past I/O data and the future inputs to predict the future outputs of a system. Then a predictive controller is parameterized and developed by the identified predictor. The SPC method skips the identification of the system model and only relies on its system operation data, which can therefore be easily implemented online to accommodate various control systems.

However, similar to the model predictive control (MPC) methods [4–9], the SPC method also generates an optimal predicted input vector and applies only the first component at every time instant to the system, which is known as the ‘receding horizon optimization’. Thus, an optimization problem is solved recursively online at every time instant to guarantee its stability and optimality, which costs considerable computation load in this process. In order to reduce the computation load, this paper introduces the event-

triggered idea into the SPC method. Conventional event-triggered methods were originally applied in the network-based control to reduce the communication load due to the limited network resources [10–13]. In recent years, some results have been published with introducing the event-triggered idea into the predictive control methods to improve the algorithm efficiency [14–18]. In [16], an event-triggered MPC scheme for constrained continuous-time nonlinear systems with bounded disturbances is developed, which can reduce the communication load with the proposed algorithm. While in [17], the stability and feasibility of the above proposed algorithm can be ensured by properly designing the prediction horizon. In [18], an event-triggered law is proposed for the closed-loop SPC method with system input and output data. The corresponding event-triggered law is designed based on the pre-designed observer parameters. Therefore, all the above event-triggered methods are designed based on either the complete system model or the equivalent identified model, which are model-based methods instead of data-driven ones.

Motivated by the above results, this paper develops a novel event-triggered SPC method for partially unknown discrete-time linear time-invariant (LTI) systems with actuator constraints. With the input-to-state (ISS) stability theory, the defined input error is checked to decide whether the input component is applied to the system or not. Without the complete system model information, the admissible state feedback law with its kernel matrix is derived by the technique of reinforcement learning method via system data. Then the proposed event-triggered law is designed based on

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them. With partial system model parameters and the system on-line data, the event-triggered SPC method can control the system with the expected performance and reduced computation load.

The rest of this paper is organized as follows: Section 2 introduces the conventional SPC method for the system with actuator constraints; Section 3 elaborates the main parts of the event-triggered SPC algorithm, including obtaining the stable feedback control parameters by the reinforcement learning method and designing the proposed event-triggered law; Section 4 illustrates the simulation results of the proposed method and verifies its satisfactory effect; Section 5 draws the conclusion and ends this paper.

## 2. Problem statement and preliminaries

Consider a discrete-time LTI system of the form:

$$x_{k+1} = Ax_k + Bu_k, \tag{1}$$

where  $x_k \in \mathbb{R}^n$  denotes the system state and  $u_k \in \mathbb{R}^m$  denotes the system input.

It is assumed that the following assumptions are satisfied for the system.

### Assumptions:

1. The system matrix  $A$  is unknown while the input matrix  $B$  is known;
2. The system actuators are limited by the constraints such that  $u_{\min} \leq u_k \leq u_{\max}$ ;
3. The system dynamic  $(A, B)$  is controllable;
4. The data of the state and input can be both obtained online during the system operation.

**Remark 1.** Assumption 1 is the description of the partially unknown system. Sometimes, due to the complicated interaction inside the system plant, the accurate parameters of the state matrix  $A$  is hard to obtain with the modelling uncertainty. However, the input matrix  $B$  can be derived based on the actuator physical structures [36]. Meanwhile, such assumption is typical in the reinforcement learning algorithms for discrete-time systems [19–21]. Assumption 2 indicates the actuator constraints due to the actuator physical conditions. Assumptions 3–4 are typical assumptions in state feedback control problems and without further explanation.

To obtain the data-based equations of system (1), the following notations are defined by a certain data vector  $\omega_k \in \mathbb{R}^\zeta$

$$\omega_r(k) = [\omega_k^T \quad \omega_{k+1}^T \cdots \omega_{k+r-1}^T]^T \in \mathbb{R}^{\zeta r} \tag{2}$$

$$\Omega_{k_1}^{k_2} = \begin{bmatrix} \omega_{k_1} & \omega_{k_1+1} & \cdots & \omega_{k_1+s-1} \\ \omega_{k_1+1} & \omega_{k_1+2} & \cdots & \omega_{k_1+s} \\ \vdots & \vdots & \ddots & \vdots \\ \omega_{k_1+r-1} & \omega_{k_1+r} & \cdots & \omega_{k_2} \end{bmatrix} \triangleq [\omega_r(k_1) \quad \omega_r(k_1+1) \quad \cdots \quad \omega_r(k_1+s-1)] \in \mathbb{R}^{\zeta r \times s}, \tag{3}$$

where  $s, r$  are some integers ( $s \gg r$ ).

The notation  $\Omega_{k_1}^{k_2}$  represents the data Hankel matrix, with  $k_1$  representing the instant of the first block element on the upper left corner of the matrix and with  $k_2$  denoting the instant of the last block element on the lower right corner of the matrix.

The typical SPC method is based on the following input-output equations of system (1):

$$X_{-j}^{-N} = \Gamma X_p + H U_{-j}^{-N} \tag{4}$$

$$X_{-j+N}^0 = \Gamma X_f + H U_{-j+N}^0, \tag{5}$$

where the subscript  $p$  denotes the *past* time interval and  $f$  denotes the *forward* time interval. Since the measurements of the inputs  $u_k$  and the states  $x_k$  are available for  $k \in \{-j, \dots, 0\}$ , the following data block Hankel matrices are constructed based on (2)–(3):

$$\begin{aligned} U_{-j}^{-N} &= [u_N(-j), u_N(-j+1), \dots, u_N(-2N+1)] \\ U_{-j+N}^0 &= [u_N(-j+N), u_N(-j+N+1), \dots, u_N(-N+1)] \\ X_{-j}^{-N} &= [x_N(-j), x_N(-j+1), \dots, x_N(-2N+1)] \end{aligned} \tag{6}$$

$$X_{-j+N}^0 = [x_N(-j+N), x_N(-j+N+1), \dots, x_N(-N+1)]$$

The past and future state sequences in (4)–(5) are defined as:

$$X_p = [x_{-j} \quad x_{-j+1} \quad \cdots \quad x_{-2N+1}] \tag{7}$$

$$X_f = [x_{-j+N} \quad x_{-j+N+1} \quad \cdots \quad x_{-N+1}] \tag{8}$$

and the extended observable matrix  $\Gamma$  and the corresponding Toeplitz matrix  $H$  are given:

$$\Gamma = \begin{bmatrix} I \\ A \\ \vdots \\ A^{N-1} \end{bmatrix} \tag{9}$$

$$H = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ B & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ A^{N-2}B & A^{N-3}B & \cdots & 0 \end{bmatrix} \tag{10}$$

where  $I$  denotes the identity matrix of appropriate dimensions.

**Remark 2.** The previous system data are collected as in (6). The persistency of excitation condition of the constructed data matrices should be satisfied to ensure the consistency of identification. Thus, the data are collect under adequately different system operation conditions and the column number in the above data matrices is much larger (typically 100 times) than the row number. Detailed explanations can be referred to [1–3].

Then in this paper, the SPC problem with actuator constraints is summarized: given a future reference state trajectory  $r_k$  for  $k \in \{1, 2, \dots, N_f\}$  and measurements of the inputs  $u_k$  and the states  $x_k$  of the system (1) for  $k \in \{-j, \dots, 0\}$ , find the input sequence  $u^{N_f} \triangleq u_{N_f}(1) = [u_1^T, \dots, u_{N_f}^T]^T$  with  $u_{\min} \leq u_i \leq u_{\max}$ ,  $i \in \{1, 2, \dots, N_f\}$ , such that the following quadratic cost function  $J$  is minimized:

$$J = \sum_{k=1}^{N_f} (\hat{x}_k - r_k)^T Q (\hat{x}_k - r_k) + u_k^T R u_k, \tag{11}$$

where  $\hat{x}_k$  is the  $k$ -step-ahead predicted state,  $Q \in \mathbb{R}^{n \times n}$  and  $R \in \mathbb{R}^{m \times m}$  are user-defined symmetric positive-definite weighing matrices.  $N_f$  denotes the future optimization horizon and is defined by the user. Then the SPC problem with actuator constraints can be solved by Algorithm 1 as follows.

**Algorithm 1** (The conventional SPC algorithm with actuator constraints).

Step 1. From the data set  $\{u_{-j}, \dots, u_0, x_{-j}, \dots, x_0\}$ , construct the data block Hankel matrices  $X_{df} \triangleq X_{-j+N}^0$ ,  $U_f \triangleq U_{-j+N}^0$  and  $W_p$  as (4)–(8), where  $W_p \triangleq [U_{-j}^{-N} \quad X_{-j}^{-N}]^T$ .

Step 2. Make the QR-decomposition

$$\begin{bmatrix} W_p \\ U_f \\ X_{df} \end{bmatrix} = \begin{bmatrix} R_{11} & 0 & 0 \\ R_{21} & R_{22} & 0 \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} Q_1^T \\ Q_2^T \\ Q_3^T \end{bmatrix} \tag{12}$$

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