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Adaptive synchronization of multiple uncertain coupled chaotic systems via sliding mode control

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ABSTRACT

This paper mainly investigates two kinds of sliding mode synchronization (SMS) for multiple chaotic systems with unknown parameters and disturbances. Both multiple coupled systems and uncoupled systems are considered. For multiple uncoupled chaotic systems (MUCSs), the sliding mode control scheme is designed to ensure that multiple response systems synchronize with one drive system under the effects of external disturbances, and the appropriate adaptive laws are proposed to estimate unknown parameters. For multiple coupled chaotic systems (MCCSs), the definition of transmission synchronization is given first. Furthermore, a special integral sliding surfaces is selected, and the corresponding controllers with the compensation terms are developed to realize SMS between every drive chaotic system and every respond system in transmission mode. Finally, some numerical examples are presented to illustrate the validity of theoretical results.

1. Introduction

In the past few years, synchronization [1] of multiple chaotic systems has attracted increasing attention, and many innovation results have been developed, which have brought about some theoretical values for multilateral communications, secret signaling and complex networks [2–4]. Many various synchronization problems have been discussed for multiple coupled or uncoupled chaotic systems. including compete synchronization [5,6], anti-synchronization[6], hybrid synchronization [7], general projective synchronization [8], combination synchronization [9], etc. However, more response systems only synchronize one drive system in almost all the aforementioned results. Recently, transmission synchronization (TS) has been widely concerned to achieve multiple chaotic systems with different initial conditions step by step [10–13]. Its core idea is based on the benefits of cluster synchronization scheme of complex drive-response networks [14], and can ensure that every system is not only the drive system, but the response system. This transmission structure has more valuable than the pervious mode, may be possible to attain vastly better performance for the security of secret signals in multilateral communications.

It is well known that unknown model uncertainties exist in most practical chaotic signal transmission process. How to tackle with uncertainties and achieve chaos synchronization have been continuously discussed by lots of scholars [12,13,15–19]. To mention a few, Chen et al. designed two adaptive control schemes to investigate two kinds of synchronization problems for multiple uncertain chaotic systems [12,13]. Sun et al. dealt with finite time combination synchronization among three or four real and complex chaotic systems and estimated unknown parameters [18,19]. However, all above mentioned works have focused on multiple uncoupled chaotic systems. From a practical point of view, it is more important to investigate chaos synchronization among multiple uncertain coupled chaotic systems. Meanwhile, there are no related results on synchronization for multiple uncoupled chaotic systems with unknown parameters and disturbances.

On the other hand, sliding mode control method [20–22], as a more popular technique, has been successfully used to deal with the uncertainties in some important research areas due to its advantages of fast dynamic response and low sensitivity to external disturbances and model uncertainties. For chaos synchronization, much more fundamental results have been reported. As a

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few examples, Aghababa et al. designed sliding mode controllers to reach synchronization between Lorenz system and Chen system by considering unknown uncertainties and disturbances [16,17,23]. Pourmahmood et al. proposed a robust adaptive sliding mode controller to achieve synchronization between two different uncertain chaotic systems [24]. Li et al. investigated the design of chattering-free sliding mode controller for chaos synchronization [25]. In [26], sliding mode synchronization for fractional-order chaotic systems were discussed. Liu et al. used adaptive sliding mode control to investigate finite time stabilization problem of uncertain chaotic systems with input nonlinearity [27]. Cai et al. gave a deep discussion on modified projective synchronization for two uncertain chaotic systems [28]. From the above mentioned results, seldom authors considered synchronization among multiple uncertain chaotic systems. In [13], the authors only discussed sliding mode synchronization of multiple uncoupled chaotic systems with uncertainties and disturbances, and more general cases were not established on multiple coupled chaotic systems with unknown parameters and disturbances. In addition, adaptive integral sliding mode control method has been applied to synchronize coupled nonlinear systems in [29–31]. Therefore, the above analysis motivate us to carry out such questions.

In response to the above discussions, the purpose of this paper is to investigate SMS of multiple uncertain chaotic systems with unknown parameters and disturbances. Both MUCSs and MCCSs are considered. The main contributions of this paper are listed: (1) On the basis of [13], sliding mode synchronization for MUCSs is continued to investigate by considering the effects of unknown parameters and disturbances from a theoretical point of view. (2) TS among multiple uncertain chaotic systems with the coupling terms is investigated for the first time. (3) For MCCSs, a special class of sliding mode controllers with the compensation terms and adaptive laws are developed to avoid the influence of the coupling terms and deal with effectively unknown parameters and disturbances.

The rest of this paper is organized as follows. Section 2 discusses SMS of MUCSs with unknown parameters and disturbances. In Section 3 sliding mode control scheme is designed to realize TS for multiple uncertain coupled chaotic systems. Some numerical examples and analysis are illustrated in Section 4 Finally, some conclusions and the future works are shown in Section 5

2. SMS of MUCSs with unknown parameters and disturbances

In this section, N uncoupled chaotic systems with unknown parameters and disturbances can be considered, where the first system is

$$\begin{cases} \dot{x}_{11}(t) = f_{11}(x_{11}(t),\dots,x_{1n}(t)) + F_{11}(x_{11}(t),\dots,x_{1n}(t))\theta_1 + d_{11}(t), \\ \dot{x}_{12}(t) = f_{12}(x_{11}(t),\dots,x_{1n}(t)) + F_{12}(x_{11}(t),\dots,x_{1n}(t))\theta_1 + d_{12}(t), \\ \vdots \\ \dot{x}_{1n}(t) = f_{1n}(x_{11}(t),\dots,x_{1n}(t)) + F_{1n}(x_{11}(t),\dots,x_{1n}(t))\theta_1 + d_{1n}(t), \end{cases} \tag{2.1}$$

where $x_{1l}(l = 1,\dots,n)$ is the state of the drive system (2.1) and $x_1(t) = [x_{11}, x_{12},\dots,x_{1n}]^T$; $f_{1l}(x_1)$ is the continuous function and $f_1(x_1(t)) = [f_{11}, f_{12},\dots,f_{1n}]^T$; $F_{1l}(x_1(t))$ is the matrix function and $F_1(x_1(t)) = [F_{11}, F_{12},\dots,F_{1n}]^T$; $\theta_1 = [\theta_{11}, \theta_{12},\dots,\theta_{1n}]^T$ is the vector of unknown parameters, and $D_1(t) = [d_{11}, d_{12},\dots,d_{1n}]^T$ is the vector of all external disturbances. The other $N-1$ systems can be expresses as the following general form,

$$\begin{cases} \dot{x}_{j1}(t) = f_{j1}(x_{j1}(t),\dots,x_{jn}(t)) + F_{j1}(x_{j1}(t),\dots,x_{jn}(t))\theta_j + d_{j1}(t) + u_{j-1,1}, \\ \dot{x}_{j2}(t) = f_{j2}(x_{j1}(t),\dots,x_{jn}(t)) + F_{j2}(x_{j1}(t),\dots,x_{jn}(t))\theta_j + d_{j2}(t) + u_{j-1,2}, \\ \vdots \\ \dot{x}_{jn}(t) = f_{jn}(x_{j1}(t),\dots,x_{jn}(t)) + F_{jn}(x_{j1}(t),\dots,x_{jn}(t))\theta_j + d_{jn}(t) + u_{j-1,n}, \end{cases} \tag{2.2}$$

where $j = 2,\dots,N$. $x_j(t) = \left[x_{j1}, x_{j2},\dots,x_{jn}\right]^T$ is the state vector of the jth system; $f_j(x_j(t)) = \left[f_{j1}, f_{j2},\dots,f_{jn}\right]^T$ is the continuous function vector; $F_j(x_j(t)) = \left[F_{j1}, F_{j2},\dots,F_{jn}\right]^T$ is the matrix function vector; the vector of the unknown parameters is $\theta_j = \left[\theta_{j1}, \theta_{j2},\dots,\theta_{jn}\right]^T$ and the external disturbance is $D_j(t) = \left[d_{j1}, d_{j2},\dots,d_{jn}\right]^T$; $u_{j-1} = \left[u_{j-1,1}, u_{j-1,2},\dots,u_{j-1,n}\right]^T$ is the vector of control inputs.

Consider α as the desired scaling factor, and it is assumed that the external disturbances are norm-bound in C^1, i.e. $|d_{1i}(t)| \le \beta_{1i}$ $(i = 1,\dots,n)$ and $|d_{ji}(t)| \le \beta_{ji}$, where β_{1i} and β_{ji} are constants. Now we choose the first chaotic system as the drive system and the other $N-1$ systems as the response systems, then the state of synchronization error can be expressed as $e_{j-1,i}(t) = x_{ji}(t) - \alpha x_{1i}(t)$, where $j = 2,\dots,N$ and $i = 1,\dots,n$, and the corresponding error system $\dot{e}_{j-1}$ can be easily obtained as

$$\begin{cases} \dot{e}_{j-11} = f_{j1}(x_j) - \alpha f_{11}(x_1) + F_{j1}(x_j)\theta_{j1} - \alpha F_{11}(x_1)\theta_{11} + d_{j1}(t) - \alpha d_{11}(t) + u_{j-1,1} \\ \dot{e}_{j-1,2} = f_{j2}(x_j) - \alpha f_{12}(x_1) + F_{j2}(x_j)\theta_{j2} - \alpha F_{12}(x_1)\theta_{12} + d_{j2}(t) - \alpha d_{12}(t) + u_{j-1,2} \\ \vdots \\ \dot{e}_{j-1,n} = f_{jn}(x_j) - \alpha f_{1n}(x_1) + F_{jn}(x_j)\theta_{jn} - \alpha F_{1n}(x_1)\theta_{1n} + d_{jn}(t) - \alpha d_{1n}(t) + u_{j-1,n} \end{cases} \tag{2.3}$$

Remark 1. From (2.3), it is easy to know that the above synchronization mode is that $N-1$ response systems will synchronize one drive system, which can be considered as the basic synchronization mode among multiple chaotic systems.

Remark 2. Synchronization of chaotic systems is unavoidably subject to external disturbances. In some actual systems, the upper bounds of disturbances are not only existing, but also the constants. From a theoretical point of view, we assume that the upper bounds β_{1i} and β_{ji} are given known constants, which follows some published cases in [13,28,32,33]. In fact, such assumptions are usual and suitable for chaos synchronization, which can facilitate our theoretical research in this paper. Meanwhile, it is pointed out that the upper bounds of most of disturbances are unknown constants in practical situations, and some discussions have been given to estimate the unknown upper bounds of disturbances for two chaotic systems [15,16,23,24]. From a more practical point of view, investigation of disturbance's effect in synchronization among multiple chaotic systems is our next goal by considering the unknown upper bounds.

Remark 3. The definition of the desired scaling factor α means that there exists projective synchronization among N chaotic systems, then it is easy to know that complete synchronization [5,6], anti-synchronization [6] and another proposed synchronization [10,12,13] can be considered as special cases in our model.

For the above synchronization problem, sliding mode control method is used to design the appropriate controllers u_{j-1} to stable the error system $\dot{e}_{j-1}$ and estimate the unknown parameters θ_1 and θ_j. First, we select a simple sliding surface as follows:

$$s_{j-1,i}(t) = \lambda_{j-1,i}e_{j-1,i}(t) \tag{2.4}$$

where $s_{j-1}(t) = [s_{j-1,1}, s_{j-1,2},\dots,s_{j-1,n}]^T$ and $\lambda_{j-1} = diag(\lambda_{j-1,1}, \lambda_{j-1,2},\dots,\lambda_{j-1,n})$, and $\lambda_{j-1,i}$ is a positive constant. Having proposed the suitable sliding surface, the next step is to design $u_{j-1,i}$ to ensure the existence of sliding motion, which can be given as

$$u_{j-1,i} = -f_{ji}(x_{ji}) + \alpha f_{1i}(x_{1i}) - F_{ji}(x_{ji})\hat{\theta}_j + \alpha F_{ji}(x_{ji})\hat{\theta}_1 - k_{j-1,i}\mathrm{sgn}(s_{j-1,i}), \tag{2.5}$$