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Distributed optimization problem for double-integrator systems with the presence of the exogenous disturbance

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ABSTRACT

The aim of this paper is to study the distributed optimization problem for continuous-time multi-agent systems with the existence and the interference of external disturbance, therein each agent is described as double-integrator dynamic. To reject the exogenous disturbance, the distributed algorithm is proposed for each agent based on the internal model principle. The proposed algorithm only utilizes the position information of each agent from its neighbors subject to the undirected graph, which can reduce communication costs and energy consumptions in applications. Moreover, the algorithm only needs the cost functions of the agent itself, which can greatly protect the privacy of other agents. The optimal solution of the problem is thus obtained with the design of Lyapunov function and the help of convex analysis, LaSallel's Invariance Principle. Finally, two numerical simulation examples and the comparison of proposed algorithm with other previous research are presented to illustrate the persuasive effectiveness of the theoretical result.

1. Introduction

In the last few years, the research on distributed optimization problem has been emerged swiftly with an extensive applications (i.e. large-scale networks, distributed estimation, source localization) [1–8]. In addition, the distributed optimization problem for multi-agent systems is ubiquitous due to the application of distributed optimization algorithm in optimal distribution for sensor networks [9], cooperative control of mobile vehicles flocking [10], power systems [11], etc. The objective of multi-agent systems is to solve the global optimization problem by the cooperation within multiple agents in a distributed manner, where the team's objective function is composed of a sum of local objective functions, each of which is known to only one agent [12–21]. Thus, one of the critical problems is how to design a distributed optimization control protocol such that the all agents can reach convergence to optimal solution based on the shared information under undirected or directed communication topology.

Existing distributed optimization control protocols for the multi-agent systems can be roughly classified into two categories, namely, first-order control protocols [12–14,16,18–28] and second-order control protocols [21,29–31]. Note that during the system operation, disturbance usually occurs, which influences the performance of the system. Disturbance can be originated from the surrounding environment, or from the system itself. Therefore, research on multi-agent systems with the presence of the disturbance is necessary, especially due to requirements of real life. However, only a few works have been presented on the distributed optimization problem with the presence of disturbance [16,27,28]. In [27], the authors have proposed a first-order distributed control protocol to solve the optimization problem for multi-agent systems with the presence of external disturbance based on internal model principle and Lyapunov-based method. In [28], the dynamic optimization problem for continuous-time first-order multi-agent systems together with the presence of disturbance with unknown frequency is investigated. Therein, by using the adaptive internal model, a distributed control protocol is proposed to obtain the optimization along with the disturbance rejected under a connected interaction topology. Recently, a first-order distributed control protocol is designed to obtain the optimization for non-linear multi-agent systems with the presence of the disturbance in [16]. In the above related literatures [16,27,28], their distributed control protocols require single-integrator for multi-agent systems.

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Beside the distributed optimization control protocols that require first-order multi-agent systems, there have been only a few papers investigating second-order control protocol to solve the optimization problem [21,29,31]. In [21], the authors have considered distributed optimization problem for continuous-time double-integrator systems by the non-smooth algorithm; in [29], a second-order control protocol for multi-agent networks has been proposed to solve the optimization problem, based on the gradient algorithm and Lyapunov function analysis; in [31], based on the projection method, the authors have solved bound-constrained distributed optimization problem with second-order multi-agent systems. More recently, [30] introduces a time-varying distributed optimization problem for second-order multi-agent systems with fixed directed communication topology. In our previous work [32], a distributed optimization control protocol for second-order multi-agent networks is proposed based on only using the position information of each agent among its neighbors. However, in these aforementioned works [21,29–32], the presence of disturbance is neglected. In addition, the control protocols proposed in [21,29–31], their control protocols require each agent to transmit both information position and velocity of each agent (i.e., x_i and $\dot{x}_i$) to its neighbors.

Motivated by [16,27,28] and the consensus control protocols in [21,29–32], a distributed optimization control protocol based on double-integrator systems in the presence of external disturbance is designed. Moreover, the disturbance reject scheme is injected into the proposed algorithm based on the internal model principle. Besides, with the proposed algorithm, each agent only needs to know its own local cost function, while does not share this privacy information even with its neighbors. Therefore, the proposed algorithm can significantly reduce communication costs and energy consumptions, as well as protect the privacy of each agent. With the design of Lyapunov function and the help of internal model principle, LaSallel's Invariance Principle, convex analysis, the optimal solution is obtained and the optimization problem is solved. Finally, two numerical examples and the comparison of proposed algorithm with existing algorithms are presented to illustrate the theoretical result. The main contributions of this paper are listed as follows:

(1) The purpose of this paper mainly focuses on solving the distributed optimization problem of double-integrator systems with the presence of external disturbance. It is well known that the disturbance is inevitable in real-time applications. The impact of disturbance in the single-integrator systems has been discussed in [16,27,28]. However, in the existing works [21,29–32] on the distributed optimization of double-integrator systems, the influence of disturbance is neglected.

(2) Comparing with [21,29–31], the proposed algorithm only utilizes the position information of each agent from its neighbors, which can reduce communication costs and energy consumptions. Moreover, the algorithm only needs the cost functions of the agent itself, which can greatly protect the privacy of other agents.

(3) By utilizing the internal model principle, a criterion on solving the distributed optimization problem of double-integrator systems is derived to guarantee obtaining the optimization with the presence of external disturbance.

The rest of this paper is organized as follows: some basic concepts and definitions and some useful lemmas are presented in Section 2; the problem statement is formulated and the existing control protocols are introduced in Section 3; the distributed optimization algorithm and the disturbance reject scheme based on the internal model principle are proposed and the convergence proof is given in Section 4; two numerical examples and the comparison of proposed algorithm with existing works are presented to illustrate the theoretical result in Section 5; the conclusion of the paper and the future works are presented in Section 6.

2. Preliminaries

In this section, some basis concepts and definitions on convex function [33], algebraic graph theory [34] and some useful lemmas on Laplace matrix are introduced.

2.1. Concepts and definitions

An undirected $\mathcal{G}=\{\upsilon,\varepsilon\}$ consists of a vertex set $\upsilon=\{1,2,\dots,N\}$ and an edge set ε. An edge $\{i,j\}\in\varepsilon$ denotes by (v_i, v_j), can receive each other's information, and this means v_i and v_j are neighbors. All neighbors of vertex v_i are denoted by $N_i=\{j:(\upsilon_j,\upsilon_i)\in\varepsilon\}$. If there is a path between any two vertices of an graph $\mathcal{G}$, then the graph is connected.

The adjacency matrix of $\mathcal{G}$ is denoted by $\mathcal{A}=[a_{ij}]\in\mathbb{R}^{N\times N}$, where $a_{ii}=0$ (no self-loop), $a_{ij}>0$ if $(v_i, v_j)\in\varepsilon$ and $a_{ij}=0$ otherwise. If $a_{ij}=a_{ji}$, then the graph $\mathcal{G}$ is called undirected.

The Laplacian matrix of graph $\mathcal{G}$ is donated as $L=[l_{ij}]\in\mathbb{R}^{N\times N}$ and l_{ij} is defined as follows:

$$\begin{cases} l_{ii}=\sum_{j=1}^{N}a_{ij}, \\ l_{ij}=-a_{ij}, i\neq j. \end{cases}$$

Definition 1 ([33]). A differentiable function $f(\cdot)$ is strictly convex on $\mathbb{R}^n\rightarrow\mathbb{R}$ if

$$(y-x)^T(\triangledown f(y)-\triangledown f(x))>0, \quad \forall x\neq y\in\mathbb{R}^n.$$

$f(\cdot)$ is a m-strongly convex (for a constant $m>0$) if $(y-x)^T(\triangledown f(y)-\triangledown f(x))>m\parallel y-x\parallel^2$ for $x\neq y$.

Definition 2 ([33]). A function $f(\cdot)$: $\mathbb{R}^n\rightarrow\mathbb{R}^n$ is Lipschitz with constant $\omega>0$, or simply ω-Lipschitz if $\parallel f(x)-f(y)\parallel\leqslant\omega\parallel x-y\parallel$, $\forall x,y\in\mathbb{R}^n$.

2.2. Useful lemmas

Lemma 1 ([34]). *The Laplacian matrix L of undirected graph $\mathcal{G}$ has the following properties:*

1. L is symmetric and positive semi-definite;

2. L has a simple zero eigenvalue and all the other eigenvalues are positive if and only if the graph $\mathcal{G}$ is connected;

3. $\lambda_1(L)=0$ and $1_N=[1,1,\dots,1]^T\in\mathbb{R}^N$ is its eigenvector, and $\lambda_2(L)>0$ if and only if the graph $\mathcal{G}$ is connected.

Lemma 2 ([35]). *For any a scalar $a>0$, vectors $x,y\in\mathbb{R}^n$, and positive definite matrix $M\in\mathbb{R}^{n\times n}$, the following inequality holds:*

$$2x^Ty\leq ax^TMx+a^{-1}y^TM^{-1}y.$$

Here, the undirected graph subjects to the following assumption:

Assumption 1. Suppose that undirected graph $\mathcal{G}=\{\upsilon,\varepsilon\}$ is connected.

3. Problem statement and related works

3.1. Problem statement

In this subsection, the distributed optimization problem for second-order multi-agent systems with the presence of disturbance is introduced.

In this paper, the optimization problem is defined as following:

$$\min_{x\in\mathbb{R}^n} f(x)=\sum_{i=1}^{N}f_i(x), x\in\mathbb{R}^n, \tag{1}$$

where $f_i(\cdot):\mathbb{R}^n\longrightarrow\mathbb{R}$ is convex objective function of agent i, which is only known by agent i, $f(\cdot)$ is defined as a sum of $f_i(\cdot)$,

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