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Short Communication

Stabilization of supply networks with transportation delay and switching topology

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ABSTRACT

This paper is concerned with the stabilization problem for a class of dynamical supply networks, where the transportation delay, uncertain demand and switching topology are all taken into account. The main purpose is to design a state feedback control strategy to stabilize such a complex system. Based on the Lyapunov stability theory, a sufficient condition for the existence of state feedback controller is obtained, which ensures the exponential stability of supply networks at the stationary states. In addition, our main results also guarantee a prescribed H_∞ disturbance attenuation level with respect to the uncertain demand. A simulation study is finally included to show the effectiveness of the proposed stabilization strategy.

1. Introduction

Supply network is a complicated dynamical system that is composed of a set of facilities, connected by transportation links transforming the raw materials or resources into intermediate and the products into the end consumers [1,2]. In the recent a few years, various supply network models have been proposed to study the supply network system [3–5]. In [6], the authors proposed a supply network that is governed by balance equations and equations for the adaptation of production speeds and studied the stability and dynamics of supply networks. In [7], a stochastic discrete-time controlled dynamical model was introduced the model the supply network, and a state-feedback control policy was obtained to control the material flow of supply network. The authors in [8] analyzed the propagation and amplification of order fluctuations (i.e., the bullwhip effect) in supply chain networks operated with linear and time-invariant inventory management policies. The supply chain network in that paper is allowed to include multiple customers (e.g., markets), any network structure, with or without sharing information.

On the other hand, many complex systems are naturally subject to time delay due to finite transportation speed [9–11]. For example, the real-world supply networks are geographically distributed and the transportation of materials among the suppliers and customers usually takes much time, which can be modeled as a large-scale system with a time-delay term [12]. In [12], the Lyapunov–Razumikhin approach was utilized for the stability analysis of supply networks with time-delay. However, the time-delay in [12] is constant, which is unrealistic in practice. Recently, the authors proposed a new model within the discrete-time singular form in [13]. In this work, they considered the local capacity control for a class of production networks of autonomous work systems with time-varying delays in the capacity changes. Attention is focused on the design of a controller gain for the local capacity adjustments that maintains the work-in-progress (WIP) in each work system in the vicinity of planned levels, and guarantees the asymptotic stability of the system and reduces the effect of the disturbance input on the controlled output to a prescribed level. The recent advances on the stability and stabilization of time-delay supply network are referred to [14,15] and the references therein.

In practice, the topology structure of supply networks is changing with the time due to interconnections changing between the suppliers, which will lead to the instability of supply networks. However, this issue has been overlooked in the literatures. Very recently, the authors in [16] proposed a discrete-time Markov chain model to characterize the unreliable production capacity in serial supply chain networks. Based on the results in [16], the authors in [17] introduced a new supply networks model with stochastic switched topology that is dependent on a continuous time Markov process and study the stabilization strategies of supply networks model with stochastic switched topology and uncertain demand. They showed that the controller gains can be obtained by solving a bilinear matrix inequality. It should be noted

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that either [16] or [17] assumes that the transition probability is known precisely in order to simplify the system analysis and design. However, the likelihood of obtaining such available knowledge is actually questionable, and the cost is considerably expensive for such a complex supply network. Now, the problem is whether we can stabilize the supply networks with topology switching without resort to the precise knowledge on the transition probabilities of Markov process. To the best of the authors' knowledge, such a challenging work has not been well investigated in the literature. This motivates the present study.

In this paper, we present a new supply network model with topology switching, time-varying transportation delay and unknown customer demand. Unlike the aforementioned work, the statistics information on the transition probability from one topology to another is completely unknown. By the Laypunov stability theory and the robust control approach, a sufficient condition for the existence of the control strategy is obtained such that the supply network is exponentially stable. In addition, the prescribed H_∞ disturbance attenuation level for dynamical supply networks with uncertain demand is also guaranteed. From the simulation studies, we see that the supply network can be stabilized by our control strategy.

Notations: The notation used throughout the paper is standard. We use W^T, and $\|W\|$ to denote respectively, the transpose, and the induced norm of any square matrix W. We use $W>0$ to denote a positive-definite matrix W and I to denote the identity matrix with appropriate dimensions. Let R^m denote the m dimensional Euclidean space. $L_2[0,+\infty)$ is the space of square integrable infinite sequence. The symbol "$*$" is used in some matrix expressions to represent the symmetric terms, and they symbol "diag{•}" stands for a block-diagonal matrix.

2. Problem formulation

Consider a supply network that consists of n suppliers, where the supplier i delivers materials or resources to other supplier j $(j \neq i)$ with a delivery rate $c_{ij}x_j(t)$, where c_{ij} is the connection weight coefficient of the supplier i and j. Let the stock level of supplier i at time t be $y_i(t)$, and the delivery rate of supplier i at time t be $x_i(t)$. Hence, the supply network can be described by a directed complex network model with the interconnection of different suppliers. It is known that the dynamics of a supply network include two parts: one is the change of inventories $y_i(t)$ and the other is the adaptation of the delivery rate $x_i(t)$ of the suppliers. On the other hand, the delivery among different suppliers usually suffers from time delay due to the finite transportation speed. Therefore, the inventory $y_i(t)$ of each supplier i is described by the following material balance equation:

$$\dot{y}_i(t) = x_i(t) - \left(\sum_{j=1}^{n} c_{ij} x_j(t-\tau(t)) + d_i(t) \right) \tag{1}$$

where $d_i(t)$ is the demand rate of the end customers for supplier i. It is assumed that the demand rate $d_i(t)$ is unknown but belongs to $L_2[0,+\infty)$. $\tau(t)$ is the time-varying transportation delay between the supplier i and supplier j, and it is assumed to be upper bounded, i.e., $0<\tau(t)\le \tau$.

For the delivery rate $x_i(t)$ of each supplier i, it is assumed that the temporal change of the delivery rate is proportional to the deviation of the actual delivery rate from the expected one, and its adaptation takes on the average time interval T. Based on this, the delivery rate $x_i(t)$ is described by the following equation:

$$\dot{x}_i(t) = \frac{1}{T}\left(F(y_i(t)) - x_i(t)\right) + u_i(t) \tag{2}$$

where T is the adaptation time interval, and $u_i(t)$ is the control strategy to be designed. The nonlinear function $F(y_i(t))$ is the expected delivery rate, which usually decreases with the increase of stock level $y_i(t)$. According to [3] and [17], the nonlinear function is taken as follows:

$$F(y_i(t)) = 1 - \frac{(\tanh(y_i(t)-y_c) + \tanh(y_c))}{2} \tag{3}$$

where y_c is the safe stock level.

According to the form of function $F(y(t))$, we have $dF(y)/dy \le 1/2$ for all $y \in R^m$. It implies that the function $F(y(t))$ satisfies the following Lipschitz condition:

$$\|F(y_1(t)) - F(y_2(t))\| \le \frac{1}{2}\|y_1(t) - y_2(t)\|, \ \forall y_1, y_2 \in R^m \tag{4}$$

In practical supply networks, the topology structure is usually changing with the time due to interconnections changing among the suppliers. Then the connection term $C = [c_{ij}]_{n\times n}$ of the supply network becomes $C_{\rho(t)} = \left[c_{ij}^{\rho(t)}\right]_{n\times n}$, where $\rho(t) : [0,\infty) \to M = \{1,2,\cdots,m\}$ is a switching signal, and m is the possible topology number. Corresponding to the switching signal $\rho(t)$, we have the switching sequence $\{x_{t0}; (i_0,t_0), \cdots (i_k,t_k), \cdots, | i_k \in M, k=0,1,\ldots\}$, which means that the i_kth subsystem is activated when $t \in [t_k, t_{k+1})$. It is seen that the switching frequency f between the time interval $[t_0, t_l)$ can be defined as $f = (N)/(t_l - t_0)$, where N is the total number of topology switchings. Without loss of generality, in this paper the initial time instant is assumed to be $t_0 = 0$.

By considering the time-varying transportation delay, topology switching and the uncertain demand in a unified framework, the following supply network model is obtained:

$$\begin{cases} \dot{y}_i(t) = x_i(t) - \left(\sum_{j=1}^{n} c_{ij}^{\rho(t)} x_j(t-\tau(t)) + d_i(t) \right), \\ \dot{x}_i(t) = \frac{1}{T}\left(F(y_i(t)) - x_i(t)\right) + u_i(t) \end{cases} \tag{5}$$

Let $\bar{y}_i$ and $\bar{x}_i$ be the stationary state values of the system (5). Define $\hat{y}_i(t) = y_i(t) - \bar{y}_i$ and $\hat{x}_i(t) = x_i(t) - \bar{x}_i$. Then by straight forward computation, we have the following system:

$$\begin{cases} \dot{\hat{y}}_i(t) = \hat{x}_i(t) - \left(\sum_{j=1}^{n} c_{ij}^{\rho(t)} \hat{x}_j(t-\tau(t)) + d_i(t) \right), \\ \dot{\hat{x}}_i(t) = \frac{1}{T}\left(\hat{F}(y_i(t)) - \hat{x}_i(t)\right) + u_i(t) \end{cases} \tag{6}$$

where $\hat{F}(y_i(t)) = F(y_i(t)) - F(\bar{y}_i(t))$. In this paper, we aim to design the controller $u_i(t) = K_{1i}\hat{y}_i(t) + K_{2i}\hat{x}_i(t)$ such that the closed-loop system is exponentially stable.

Denote

$$\hat{y}(t) = [\hat{y}_1(t), \hat{y}_2(t), \cdots, \hat{y}_n(t)]^T, \hat{x}(t) = [\hat{x}_1(t), \hat{x}_2(t), \cdots, \hat{x}_n(t)]^T,$$
$$w(t) = [d_1(t), d_2(t), \cdots, d_n(t)]^T, u(t) = [u_1(t), u_2(t), \cdots, u_n(t)]^T,$$
$$C_{\rho(t)} = \begin{bmatrix} 0 & -c_{12}^{\rho(t)} & \cdots & -c_{1n}^{\rho(t)} \\ -c_{21}^{\rho(t)} & 0 & \cdots & -c_{2n}^{\rho(t)} \\ \vdots & \vdots & \ddots & \vdots \\ -c_{n1}^{\rho(t)} & -c_{n2}^{\rho(t)} & \cdots & 0 \end{bmatrix},$$
$$G = \mathrm{diag}\left\{\frac{1}{T}, \frac{1}{T}, \cdots, \frac{1}{T}\right\}, K_1 = \mathrm{diag}\{K_{11}, K_{12}, \cdots, K_{1n}\},$$
$$K_2 = \mathrm{diag}\{K_{21}, K_{22}, \cdots, K_{2n}\}.$$

Then, we have

$$\begin{cases} \dot{\hat{y}}(t) = \hat{x}(t) + C_{\rho(t)}\hat{x}(t-\tau(t)) + w(t), \\ \dot{\hat{x}}(t) = G\left(\hat{F}(y(t)) - \hat{x}(t)\right) + K_1\hat{y}(t) + K_2\hat{x}(t) \end{cases} \tag{7}$$

By further manipulation, the following closed-loop supply network system is obtained:

$$\dot{z}(t) = (A + HK)z(t) + A_{di}z(t-\tau(t)) + B\hat{F}(y(t)) + Dw(t) \tag{8}$$