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A new game model for distributed optimization problems with directed communication topologies ☆

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ABSTRACT

In this paper, the distributed optimization problems of multi-agent systems with directed communication topologies are investigated by using the game theory. In particular, a new general non-cooperative game model, termed state based weakly acyclic game, is provided to solve the problem. Based on this approach, the desired global objective is achieved by designing local objective function for individual agent to make coordination decisions. It is worth noting that all the obtained equilibria are thus solutions to the proposed distributed optimization problems with directed and time-varying communication topologies. Simulations on consensus problem in multi-agent systems are provided to verify the validness of the proposed methodology.

1. Introduction

In recent years, there has been surge of research interests aimed at designing local control laws for agents to accomplish a global objective in multi-agent systems. This is partly due to broad application in many practical applications ranging from formation control [1], distributed sensor networks [2], robotics [3] and autonomous vehicles [4]. In the past decades, numerous studies have been conducted on the topics [1–9]. However, there are some underlying challenges for system designers to design a systematic decision making architecture for multi-agent systems. Such challenges include coordinating behaviors of selfish and rational agents as well as dealing with overlapping and distributed information for a potentially large number of interacting agents. Interestingly, these challenges are fitting into the category of non-cooperative game theory [10].

Recently, the appeal of applying game theoretical methodology to design and control multi-agent system is receiving significant attention [11–13]. Utilizing game theory for this purpose requires two steps. First is the game model design. It includes defining a set of choices and a local objective function for each agent. Individual agent is viewed as a self-interested decision maker and is capable of making rational decisions to optimize its local objective function. And the concept of equilibrium is used to represent the desired cooperative behaviors at which there is no incentive for any agent to unilaterally deviate. Second is the distributed learning algorithm design. It enables the agents to reach a desirable equilibrium of the designed game model through repeated interactions with each other.

Suppose the system level objective of multi-agent systems can be captured by a global objective function, then the control of multi-agent systems can be transformed to a distributed optimization problem. Game theory provides a set of powerful tools in solving the optimization problems. For example, by using some specific form of game models, such as the potential game, Marden et al. [11] designed local objective functions for agents to make coordination decisions to optimize the potential function, which captures the global objective of multi-agent systems. Arslan et al. [12] provided a potential game formulation for the autonomous vehicle-target assignment problems and sought the optimization of a global objective function through rational agents, which were capable of making decisions to optimize their own local objective functions. Furthermore, in order to ensure efficiency of the resulting equilibrium in the framework of potential games, a variety of schemes, such as the introduction of pricing or state space, have been introduced into potential games [13,16,20,23]. As a result, a larger class of state based potential game models [13,23] and distributed learning algorithms [13,20] are investigated.

However, the game models such as the potential games or state based potential games [11–13,23] are still not broad enough to meet a set of challenges because of several inherent limitations. One of such limitations is that potential game model requires the information among agents must be undirected. It is because in the potential game, if an agent ultimately changes its action, the

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changes of its local objective function must be aligned with the change of potential function. We will show that it is theoretically impossible to model the systems with directed information exchange as a potential game or state based potential game in Subsection 3.4. This inherent limitation provides the analytical justification for moving beyond potential games or state based potential games to games of a broader structure.

In this paper, a new game model, termed state based weakly acyclic games, where an underlying state space is introduced into the framework of weakly acyclic games [11,21,22], is developed to solve distributed optimization problem with directed communication topologies. The new game model possesses several advantages. First, different from the potential game, the new game relaxes the alignment requirement between the local objective function and potential function. It just requires that there exists at least one agent such that the change of its local objective function should be aligned with the potential of the game. This flexibility in the alignment requirement is important because it will relax the structural requirements on interaction topology. As a result, the requirement on the interaction is no longer undirected necessarily, and the new game model can be used to handle directed information exchange among agents. For example, it can be used in leader-following scenarios, where some agents may only have transceivers and others only have receivers [9]. Second, in the state based weakly acyclic games, the state is introduced into the weakly acyclic games. The underlying state can be exploited to provide dynamics for equilibrium improvement by encouraging agents to share limited information and use the information more efficiently, rather than aggressively compete in the pure non-cooperative games. Regardless of the above-mentioned facts, the flexibility in the alignment requirement and the introduction of state provide the system designer with additional degrees of freedom to help coordinate system behaviors. By introducing the distributed learning algorithm [10,11,17,18,20,21,26], i.e., the better reply with inertia dynamics, the state based weakly acyclic game model converges almost surely to the equilibrium which is the optimal solution to the distributed optimization problems.

The rest of the paper is organized as follows. In Section 2, the problem formulation of game model design for the distributed optimization problems is presented. In Section 3, some preliminaries about interaction topologies and game theory are introduced. In Section 4, the state based weakly acyclic game is developed and the properties of the game model are investigated. In Section 5, a distributed learning algorithm for the state based weakly acyclic game is developed. In Section 6, a simple example is provided to illustrate the methodology and some concluding remarks are given in Section 7.

2. Problem setup

Suppose the multi-agent system consists of $n \geq 2$ agents which are denoted by the set $N = \{1, 2, \ldots, n\}$ and the information flow among multiple agents is directed. Each agent $i \subseteq N$ is endowed with a set of possible decisions (or values) denoted by A_i, which is a nonempty convex subset of R. A specific joint decision profile is denoted by a vector $v \triangleq \{v_1, v_2, \ldots, v_n\}$ and $v \subseteq V$, where $V \triangleq \prod_{i \in N} V_i$ is a closed, convex and non-empty set consisting of all possible joint decisions for all agents. Suppose the system level objective of multi-agent systems can be captured by a smooth and convex global objective function $\phi : V \to R$ that the system designer seeks to minimize, and the global objective of multi-agent system can be distributed across the agents which are individually capable of making rational decisions to optimize their own local objective functions. Accordingly, we can transform the problem of control of multi-agent systems to a distributed optimization problem. Specifically, the distributed optimization problem for the multi-agent cooperative control problem takes on the general form:

$$\min_{v_i} \quad \phi(v_1, v_2, \ldots, v_n)$$
$$\text{s.t.} \quad v_i \in V_i, \quad \forall i \in N \tag{1}$$

Different from the traditional and well-known distributed optimization algorithms [24,25], in this paper, we focuses on establishing a game framework to solve the distributed optimization problem. And there are several advantages to use game theory to model and counteract selfish behaviors in distributed systems. First, game theory offers a powerful tool set to analyze interactions between such rational agents who can observe and react to their environment. Second, although agents would like to cooperate, it might be impractical or impossible to exchange the information required to implement any of the distributed optimization techniques described so far. In these cases, we use non-cooperation game to enable agents to react to limited network information and optimize their local objectives. Third, game theory provides a way to predict, analyze or even to improve the equilibrium of a non-cooperative interaction. Last but not least, the most advantage of game theory is that it provides a decomposition between the game model design and distributed learning algorithm design [13]. This decomposition could be instrumental in shifting research attention to develop methodologies for designing games for distributed optimization problems.

In the game formulation, each agent $i \subseteq N$ is modeled as a game player with its local objective function $U_i : \prod_{j \in N_i} V_j \to R$ and chooses its decision v_i in response to directed and local information. Here N_i denotes the set of agents which have directed information flow with agent i. Then each agent follows a specific distributed learning algorithm to produce a sequence of decisions $v(0), v(1), \ldots$ at each iteration $t \in 0, 1, \ldots$. Our goal is to establish local objective function for each agent in game model and the distributed learning algorithm such that the collective decisions converge to a joint decision $v^* \triangleq \{v_1^*, v_2^*, \ldots, v_n^*\}$ that solves the distributed optimization problem in (1). Next, we will give a simple example to motivate the theoretical development in this paper.

2.1. An illustrative example

Consider a distributed optimization formulation of the consensus problem by designing local objective functions for agents in game theory environment. Suppose there are n agents seeking to come to consensus by repeatedly interacting with each other. Let $N = \{1, 2, \ldots, n\}$ denotes the finite agent set and each agent i has a finite decision set $A_i \in R$. First, the goal of the consensus problem is captured by the global objective function $\varphi : A \to R$:

$$\varphi(a) = 0.5 \cdot \sum_{i \in N} \sum_{j \in N_i} (a_i - a_j)^2, \tag{2}$$

where $a_i \in A_i$ is a specific decision of agent i, $A = \prod_{i \in N} A_i$ is the set of joint decisions of all agents and N_i is the neighbor set of agent i. In the case the interaction topology among the agents is undirected and connected, the global objective function achieves the minimal value of 0 if and only if decision $a \in A$ constitutes consensus, i.e.

$$\varphi(a) = 0 \Leftrightarrow a_1 = \cdots = a_n. \tag{3}$$

Next, we will design local objective functions for agents which produce decision for agent in game theory environment. If each agent is capable of observing the decisions of all agents to formulate its own decisions, we can assign the local objective

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