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# Synchronization analysis for static neural networks with hybrid couplings and time delays ☆

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ABSTRACT

This paper deals with the synchronization problem for delayed static neural networks with hybrid couplings. When the static neural networks are affected by hybrid couplings, it is hard to deal with a large number of highly interconnected dynamical units in such a complex system. In order to solve this complicated problem, a new method is proposed to deal with the Kronecker product, and to make the synchronization problem to be easily analyzed. Further, based on the obtained result, by using the augmented Lyapunov–Krasovskii functional (LKF) method, multitude Kronecker product terms can be handled, which can introduce more relaxed conditions by employing the new type of augmented matrices with the Kronecker product operation. Finally, a numerical example is provided to demonstrate the effectiveness of the proposed synchronization scheme.



## 1. Introduction

In the past decades, complex networks have been gaining increasing research attention due to their potential applications to many real-world systems in various fields of science and engineering [1,2]. One of the most significant and interesting phenomena in complex networks is the synchronization of all dynamical nodes. Many real world problems are closely related to the network synchronization, for example the synchronous phenomena on the internet, synchronous transfer of digital or analog signals in the communication networks, and so on [3,4]. Thus, synchronization in complex networks has drawn significant research interest in recent years; see, e.g., [3–9] and the references therein.

As a special kind of complex networks, coupled neural networks provide a large class of models that can be used to describe coupled systems with continuous time and state values, as well as discrete spatial states in many research fields [10]. The dynamical behavior of coupled networks is governed by the following two mechanisms: the intrinsic nonlinear dynamics of the neural network at each node and the diffusion due to the spatial coupling among nodes. They have been investigated as theoretical models of spatio-temporal phenomena of complex networks [11]. In the past few years, the synchronization problems in coupled dynamic networks have been widely investigated due to its applications in secure communication and signal generators design [12–18]. For example, [12] presented criteria for local and global synchronization of linearly coupled dynamical systems. In [13], the authors investigated the synchronization of coupled neural networks with time-varying coupling configuration. It is well-known that time delays occur commonly in neural networks because of the network traffic congestion as well as the finite speed of signal transmission over the links. So the synchronization study of coupled neural networks with time delays is quite important. There are also many papers dealing with this issue [14–18]. For example, in [14], by introducing a novel augmented LKF method, the authors obtained sufficient conditions in terms of LMI for global synchronization of hybrid coupling neural networks with interval delay. In [16], the global stability of synchronization manifold is investigated for an array of coupled neural networks with random coupling strengths and time-varying delays.

As was shown in [19], according to whether the neuron states or local field states of the neurons are chosen as basic variables to describe the evolution rule, neural networks can be classified as local field neural networks (LFNN) and static neural networks (SNN). However, most of the existing results are concerned with the synchronization problems of coupled LFNN, there is no result on the coupled SNN yet. Therefore, the synchronization problem for such systems still has much room for further research. Motivated by the above discussions, we investigate the synchronization problem of a general SNN with hybrid couplings and time

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delays. By constructing a novel Lyapunov functional and introducing several new lemmas, some sufficient conditions are obtained to analyze the synchronization problem for coupled static neural networks. The model is novel, and the method has not been reported in the existing results for solving synchronization problem of coupled neural networks or complex networks. Finally, a numerical example is provided to illustrate the effectiveness of our methods.

Notations: $R^n$ is the $n$-dimensional Euclidean space; $R^{m\times n}$ denotes the set of $m\times n$ real matrices. $X\geq 0$ $(X>0)$ means that $X$ is a positive semidefinite (positive definite). $I_n$ represents the $n$-dimensional identity matrix. $diag(\cdot)$ denotes a block-diagonal matrix. $\left[\begin{smallmatrix} X & Y \\ * & Z\end{smallmatrix}\right]$ stands for $\left[\begin{smallmatrix} X & Y \\ Y^T & Z\end{smallmatrix}\right]$. Matrix dimensions, if not explicitly stated, are assumed to be compatible for algebraic operations.

## 2. Problem formulation and preliminaries

In this section, the problem of global asymptotic stability of neural networks with time delay is proposed. Consider the following coupled neural networks with time delays described by

$$\begin{aligned}\dot{x}_i(t) &= -Cx_i(t)+f(Ax_i(t))+f(Bx_i(t-\tau))\\ &\quad+\sum_{j=1}^{N} G_{ij}^{(1)}D_1x_j(t)+\sum_{j=1}^{N} G_{ij}^{(2)}D_2x_j(t-\tau),\quad i=1,2,\dots,N \end{aligned}\tag{1}$$

where $x_i(t)=(x_{i1}(t),x_{i2}(t),\dots,x_{in}(t))^T\in R^n$ is the neuron state vector of the $i$th network at time $t$. $f(x_i(t))=(f_1(x_{i1}(t)),f_2(x_{i2}(t)),\dots,f_n(x_{in}(t)))^T$ is the neuron activation function, $\tau>0$ denotes the transmission delay. $C=diag(c_1,c_2,\dots,c_n)>0$ is the state feedback coefficient matrix, $A$ , $B\in R^{n\times n}$ represent the connection weight matrices, $G^{(q)}=(G_{ij}^{(q)})_{N\times N}$, $(q=1,2)$ represent the coupling connections; $D_1,D_2\in R^{n\times n}$ represent the inner coupling matrix and the discrete-delay inner coupling matrix.

For simplicity, let

$$x(t)=(x_1^T(t),x_2^T(t),\dots,x_N^T(t))^T,$$

$$F(x(t))=(f^T(x_1(t)),f^T(x_2(t)),\dots,f^T(x_N(t)))^T.$$

Combining with the sign $\otimes$ of Kronecker product, model (1) can be rewritten as

$$\begin{aligned}\dot{x}(t) &= -(I_N\otimes C)x(t)+F((I_N\otimes A)x(t))+F((I_N\otimes B)x(t-\tau))\\ &\quad+(G^{(1)}\otimes D_1)x(t)+(G^{(2)}\otimes D_2)x(t-\tau)\end{aligned}\tag{2}$$

Throughout this paper, the following assumptions are needed.

**Assumption 1.** The outer-coupling configuration matrices of the complex networks satisfy

$$\begin{cases} G_{ij}^{(q)}=G_{ji}^{(q)}\geq 0, & i\neq j, q=1,2,\\ G_{ii}^{(q)}=-\sum\limits_{j=1,j\neq i}^{N} G_{ij}^{(q)}, & i,j=1,2,\dots,N\end{cases}$$

**Assumption 2** (*Liu et al. [20]*). For any $x_1,x_2\in R$ and any constants $\sigma_r^-,\sigma_r^+$, the active function satisfies

$$\sigma_r^-\leq\frac{f_r(x_1)-f_r(x_2)}{x_1-x_2}\leq\sigma_r^+,\quad r=1,2,\dots,n$$

We denote

$$\Delta_1=diag(\sigma_1^+\sigma_1^-,\dots,\sigma_n^+\sigma_n^-),$$

$$\Delta_2=diag\left(\frac{\sigma_1^++\sigma_1^-}{2},\dots,\frac{\sigma_n^++\sigma_n^-}{2}\right).$$

Next, we give some useful definition and lemmas.

**Definition 1.** Model (1) is said to be globally synchronized for any initial conditions $\Pi_{i0}(s)$ $(i=1,2,\dots,N)$, if the following holds:

$$\lim_{t\to+\infty}\|x_i(t)-x_j(t)\|=0,\qquad\forall i,\ j=1,2,\dots,N,$$

in which $\|\cdot\|$ stands for the Euclidean norm.

**Lemma 1** (*Gu et al. [21]*). *For any constant matrix $\Re\in R^{n\times n}$, $\Re^T=\Re>0$, scalar $\rho>0$ and vector function $\varpi:[0,\rho]\to R^n$, one has:*

$$\rho\int_0^\rho\varpi^T(s)\Re\varpi(s)\,ds\geq\left(\int_0^\rho\varpi(s)\,ds\right)^T\Re\left(\int_0^\rho\varpi(s)\,ds\right)$$

**Lemma 2.** *According to [20] and Assumption 2, for any diagonal matrix $J>0$, $L>0$ and constant matrix $M$ with appropriate dimensions, it follows that*

$$\begin{aligned}&\begin{bmatrix} Mx_i(t)-Mx_j(t)\\ f(Mx_i(t))-f(Mx_j(t))\end{bmatrix}^T\begin{bmatrix}-J\Delta_1 & J\Delta_2\\ * & -J\end{bmatrix}\begin{bmatrix} Mx_i(t)-Mx_j(t)\\ f(Mx_i(t))-f(Mx_j(t))\end{bmatrix}\\ &+\begin{bmatrix} Mx_i(t-\tau)-Mx_j(t-\tau)\\ f(Mx_i(t-\tau))-f(Mx_j(t-\tau))\end{bmatrix}^T\begin{bmatrix}-L\Delta_1 & L\Delta_2\\ * & -L\end{bmatrix}\\ &\times\begin{bmatrix} Mx_i(t-\tau)-Mx_j(t-\tau)\\ f(Mx_i(t-\tau))-f(Mx_j(t-\tau))\end{bmatrix}\geq 0\end{aligned}\tag{3}$$

*It is equivalent to the following equation*:

$$\begin{aligned}&\begin{bmatrix} x_i(t)-x_j(t)\\ f(Mx_i(t))-f(Mx_j(t))\end{bmatrix}^T\begin{bmatrix}-M^TJ\Delta_1M & M^TJ\Delta_2\\ * & -J\end{bmatrix}\\ &\begin{bmatrix} x_i(t)-x_j(t)\\ f(Mx_i(t))-f(Mx_j(t))\end{bmatrix}\\ &+\begin{bmatrix} x_i(t-\tau)-x_j(t-\tau)\\ f(Mx_i(t-\tau))-f(Mx_j(t-\tau))\end{bmatrix}^T\begin{bmatrix}-M^TL\Delta_1M & M^TL\Delta_2\\ * & -L\end{bmatrix}\\ &\times\begin{bmatrix} x_i(t-\tau)-x_j(t-\tau)\\ f(Mx_i(t-\tau))-f(Mx_j(t-\tau))\end{bmatrix}\geq 0\end{aligned}\tag{4}$$

**Lemma 3.** *Let $\otimes$ denote the notation of Kronecker product. Then, the following relationships hold:*

(1) $(\alpha A)\otimes B=A\otimes(\alpha B)$
(2) $(A+B)\otimes C=A\otimes C+B\otimes C$
(3) $(A\otimes B)(C\otimes D)=(AC)\otimes(BD)$

**Lemma 4** (*Liu et al. [20]*). *Let $e=(1,1,\dots,1)^T$, $E_N=ee^T$, and $U=NI_N-E_N$, $P\in R^{n\times n}$, $x=(x_1^T,x_2^T,\dots,x_N^T)^T$, and $y=(y_1^T,y_2^T,\dots,y_N^T)^T$ with $x_k,y_k\in R^n$, $(k=1,2,\dots,N)$, then*

$$x^T(U\otimes P)y=\sum_{1\leq i<j\leq N}^{N}(x_i-x_j)^TP(y_i-y_j)$$

**Lemma 5** (*Zhang et al. [14]*). *For $P_{\theta\nu}\in R^{n\times n}$ $(1\leq\theta\leq l,1\leq\nu\leq l)$, $x^{(\omega)}=((x_1^{(\omega)})^T,(x_2^{(\omega)})^T,\dots,(x_N^{(\omega)})^T)^T$, and $y^{(\omega)}=((y_1^{(\omega)})^T,(y_2^{(\omega)})^T,\dots,(y_N^{(\omega)})^T)^T$ with $x_k^{(\omega)},y_k^{(\omega)}\in R^n$ $(k=1,2,\dots,N,\ \omega=1,2,\dots,l)$, $e=(1,1,\dots,1)^T$, $E_N=ee^T$, and $U=NI_N-E_N$, the following function can be obtained:*

$$\begin{aligned}&[(x^{(1)})^T,\dots,(x^{(l)})^T]\begin{bmatrix}U\otimes P_{11} & \dots & U\otimes P_{1l}\\ \vdots & \ddots & \vdots\\ U\otimes P_{l1} & \dots & U\otimes P_{ll}\end{bmatrix}\begin{bmatrix}y^{(1)}\\ \vdots\\ y^{(l)}\end{bmatrix}\\ &=\sum_{1\leq i<j\leq N}^{N}\begin{bmatrix}x_i^{(1)}-x_j^{(1)}\\ \vdots\\ x_i^{(l)}-x_j^{(l)}\end{bmatrix}^T\begin{bmatrix}P_{11} & \dots & P_{1l}\\ \vdots & \ddots & \vdots\\ P_{l1} & \dots & P_{ll}\end{bmatrix}\begin{bmatrix}y_i^{(1)}-y_j^{(1)}\\ \vdots\\ y_i^{(l)}-y_j^{(l)}\end{bmatrix}\end{aligned}$$