Neurocomputing 148 (2015) 318–325

Contents lists available at ScienceDirect

Neurocomputing

journal homepage: www.elsevier.com/locate/neucom

Identification of nonlinear discrete systems by a state-space recurrent neurofuzzy network with a convergent algorithm

Marcos A. González-Olvera [a,*], Yu Tang [b]

[a] Universidad Autónoma de la Ciudad de México, Colegio de Ciencia y Tecnología, Mexico City, Mexico
[b] Facultad de Ingeniería, Universidad Nacional Autońoma de México, Mexico City, Mexico

ARTICLE INFO

Article history:
Received 13 December 2013
Received in revised form
25 March 2014
Accepted 28 June 2014
Communicated by M.-J. Er
Available online 17 July 2014

Keywords:
Neural-network models
Fuzzy modeling
System identification
Discrete-time systems

ABSTRACT

Recurrent neurofuzzy networks have proven to be useful in identification of systems with unknown dynamics when only input–output information is available. However, training algorithms for these structures usually require also the measurement of the actual states of the system in order to obtain a convergent algorithm and then obtain a scheme to approximate its dynamic behavior. When states are not available and only input–output information can be obtained, the stability of the training algorithm of the recurrent networks is hard to establish, as the dynamics is driven by the internal recurrent dynamics of each connection. In this paper, we present a structure and an ultimately stable training algorithm inspired by adaptive observer for black-box identification based on state-space recurrent neural networks for a class of dynamic nonlinear systems in discrete-time. The network catches the dynamics of the unknown plant and jointly identifies its parameters using only output measurements, with ultimately bounded identification and parameter error. Numerical examples using simulated and experimental systems are included to illustrate the effectiveness of the proposed method.

1. Introduction

In the literature, neural networks and fuzzy systems have been intensively used as nonlinear static identifiers, based on their approximation capabilities and reported training algorithms for control, classification, clustering and identification problems [1–4]. Moreover, their combination into *neurofuzzy networks* has proved to be a valuable architecture where the resultant structure can be trained as a neural network, and the obtained parameters can be interpreted [5].

When feedback connections occur within the structure, then dynamic systems can be identified as well, resulting in the so-called *dynamical* or *recurrent neural networks* (RNN) or simply *recurrent networks*. When a neurofuzzy structure is considered, they are called *recurrent neurofuzzy networks* (RNFN). One of the very first approaches was given by [6] for learning trajectories. Since then, recurrent networks have been reported with a wide variety of complexity.

In earlier works, networks with feedback within the internal layers for identification and control of nonlinear systems have been proposed [4]. In [7] time-delay networks are reported and used for identification, and [8] proposed fully recurrent networks, where all layers are considered to be inputs. Networks such as globally static-locally recurrent structures have been proposed by [9,10] for identification, using neurons with linear filters in the synapsis.

Algorithms such as Backpropagation-through-time [11] and Real-time recurrent learning [8] have been used for network training. However, these algorithms and structures tend to be slow and lack stability analysis [12]. Training based on linear approximations such as recursive-least-squares [13,14] and Kalman filters [15] have been reported and successfully applied. However, the Kalman filter is known to have a difficult stability analysis, so ellipsoidal methods have been reported instead [16]. Relying on the specific structure of the networks, [17,18,12] guarantee stable training with a state-space structure, but need the measurements of the actual states of the system, which may not be available in some situations.

In this work, we propose a recurrent discrete-time neurofuzzy network in state-space representation derived from the work [19,20] that is in continuous-time. Motivated from research works in adaptive observer design [21–23], the proposed network is useful to model and identify a class of nonlinear systems using only output measurements. The structure takes the advantage of the approximation capabilities of neurofuzzy networks, and the

* Corresponding author.
E-mail addresses: marcos.angel.gonzalez@uacm.edu.mx (M.A. González-Olvera), tang@unam.mx (Y. Tang).

http://dx.doi.org/10.1016/j.neucom.2014.06.066
0925-2312/

training algorithm is based on nonlinear adaptive observers. To the best of the authors' knowledge, few work has relied on the actual conditions in which a system can be actually identified *via* input–output measurements. Generally it is assumed such identification scheme to be fulfilled and to exist, but the implications are not further investigated. In this work we propose to take advantage of the controllability and observability conditions needed to assume that the input–output dynamics are contained in these signals to propose a State-space Recurrent Neurofuzzy Network (SRNN). The resulting network and the training algorithm can estimate equivalent *states* of the system, in parallel with the parameter identification, giving the required conditions for parameter convergence.

The rest of the paper is organized as follows: in Section 2 the basic analysis of the problem and the class of systems to identify. In Section 3 we present an identifier structure that trains and identifies the input–output response of a class of nonlinear systems exploiting controllability and observability conditions. In this same section the boundness and convergence of the training algorithm are discussed. Examples of the performance of the neural identifier are shown in Section 4 by identifying a nonlinear system and an experimental system. Finally, conclusions are drawn in Section 5.

2. Problem statement

In the first place, consider a single-input–single-output system in continuous time to be identified, defined by a mapping

$$y(t) = H(u(t)) \tag{1}$$

given by the state-space equation

$$\dot{\mathbf{x}} = f(\mathbf{x}(t), u(t)), \quad y(t) = h(\mathbf{x}(t)), \tag{2}$$

where $f : \Re^n \times \Re \rightarrow \Re^n$ and $h : \Re^n \rightarrow \Re$ are smooth and bounded functions, $\mathbf{x} \in \Re^n$ and $y, u \in \Re$ are the state and input–output signals respectively. The problem to be addressed in this work is to identify the input–output dynamics of the system (2) and reconstruct them when little information of functions f and h is available and only input–output information is available from the mapping $y(t) = \hat{H}(u(t))$, while $(\mathbf{x}, \mathbf{u})$ remains within a certain ball $\Omega = \{(\mathbf{x}, \mathbf{u}) | \, \|\mathbf{x} - \mathbf{x}_0\| \le \Omega_x, \ \|u - u_0\| \le \Omega_u, \} \subset \Re^{n+1}$ with center $(\mathbf{x}_0, u_0)$, as depicted in Fig. 1.

If the signals from the system (2), measured in an experimental framework, are obtained with uniform sampling $t = kT_s$, with T_s the sampling time, then a discrete-time representation of the system (2) in the form can be expressed as

$$\begin{aligned} \mathbf{x}[k+1] &= f_d(\mathbf{x}[k], u[k]), \\ y[k] &= h_d(\mathbf{x}[k]), \end{aligned} \tag{3}$$

where $f_d(\mathbf{x}[k], u[k]) = x[k] + T_s f(\mathbf{x}[k], u[k])$, $h_d(\mathbf{x}[k]) = h(\mathbf{x}[k])$.

In order to reconstruct the dynamics of this system, not the states themselves, in the first place it is necessary that the input $u(t)$ does affect the whole state $\mathbf{x}(t)$, *i.e.* that the system is controllable in the region of interest. In the second place, as the dynamics are contained in the states, these must be reflected and reconstructed from the measurement of the output $y(t)$ and $u(t)$, *i.e.* that the system is observable.

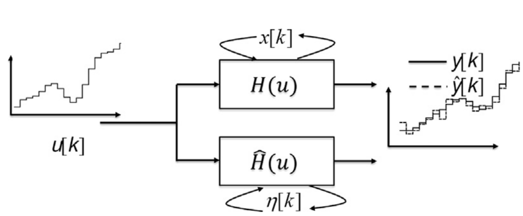

Fig. 1. Discrete-time identification process.

So, the following assumptions are necessary in order to identify the input–output dynamics $H(\cdot)$ of the system:

Assumption 1. The system H in Ω:

(i) Is controllable and observable.
(ii) Is input-to-state stable in $(\mathbf{x}, \mathbf{u}) \in \Omega$.
(iii) f and h $\in \mathcal{C}(\Omega)$.[1]

So, when (3) complies with Assumption 1, then it belongs to the class of nonlinear systems transformable to an output feedback form *via* a diffeomorphism $\mathbf{z} = T(\mathbf{x})$ [22,24–26] into

$$\begin{aligned} \mathbf{z}[k+1] &= A\mathbf{z}[k] + \overline{f}(y[k], u[k]), \\ y &= C\mathbf{z}, \end{aligned} \tag{4}$$

with $\overline{f}(y, u)$ smooth and (A, C) in observer canonical form

$$A = \begin{bmatrix} \mathbf{0}_{n\times 1} & \begin{matrix} I_{(n-1)\times(n-1)} \\ \mathbf{0}_{1\times(n-1)} \end{matrix} \end{bmatrix}, \quad C = [1 \ \ \mathbf{0}_{1\times(n-1)}]. \tag{5}$$

Notice that it is assured that the pair (A, C) is observable and with a given form. As indicated in [22], T is fairly complex to obtain; however, in the input–output identification presented in this work, due to the controllability and observability conditions required in the input–output identification problem, it is only required to exist, not to be known or calculated.

Now, using the transformed system (4), if an output injection Ly, $L \in \Re^{n\times 1}$ is considered in the state equation of (4),

$$\begin{aligned} \mathbf{z}[k+1] &= A\mathbf{z}[k] + \overline{f}(y[k], u[k]) + Ly[k] - Ly[k] \\ &= \overline{A}\mathbf{z}[k] + \overline{f}(y[k], u[k]) + Ly[k], = \overline{A}\mathbf{z}[k] + \overline{f}^*(y[k], u[k]), \\ y &= C\mathbf{z} \end{aligned} \tag{6}$$

then some designed stable dynamics can be set into matrix $\overline{A} \triangleq A - LC$, and the problem is now to identify the term $\overline{f}^*(y[k], u[k])$.

By the universal approximation theorem [27], for $\overline{f}^*$ smooth and bounded, then a neurofuzzy structure $\varphi(y[k], u[k], \theta)$ can identify it with arbitrary precision. In this particular case, we consider that the network is linearly parametrizable, so $\varphi(y[k], u[k], \theta) = \varphi(y[k], u[k])\theta$. In this way, the main objective is to find a parameter vector θ^* such that

$$\left\| \overline{f}^*(y, u) - \varphi(y[k], u[k], \theta) \right\| < \varepsilon_\eta$$

where ε_η is the approximation error of the network, given the capability of approximation given by [1]. In this sense, the objective is now to find a discrete-time recurrent neural network described by

$$\eta[k+1] = \overline{A}\eta[k] + \varphi(y[k], u[k])\theta_\eta, \quad \hat{y}[k] = C\eta[k], \tag{7}$$

such that, by using only the input signal $u[k]$ and output measurement $y[k]$, it can generate a dynamic mapping $u[k] \rightarrow \hat{y}[k]$, such that $\sup_{t \ge 0} |\hat{y}[k] - y[k]|$ is minimized by tuning the network parameter θ_η. Note that the matrix $\overline{A}$ is now defined as a design matrix $\overline{A} \triangleq A - LC$,

Remark 1. One of the main advantages of the structure (7) is that it is only necessary to identify one function $\varphi(\cdot, \cdot)$ that depends on (y, u), rather than identifying f and h in (3).

Now, given this structure, it is necessary to obtain an algorithm and a neurofuzzy network that uses input–output measurements, identifies the parameters θ_η and estimates the states $\hat{\eta}[k]$ as well

[1] Here $\mathcal{C}(\Omega)$ represents the set of continuous functions.