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Exponential synchronization of Markovian jumping neural networks with partly unknown transition probabilities via stochastic sampled-data control

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ABSTRACT

This paper investigates the exponential synchronization for a class of delayed neural networks with Markovian jumping parameters and time varying delays. The considered transition probabilities are assumed to be partially unknown. In addition, the sampling period is assumed to be time-varying that switches between two different values in a random way with given probability. Several delay-dependent synchronization criteria have been derived to guarantee the exponential stability of the error systems and the master systems are stochastically synchronized with the slave systems. By introducing an improved Lyapunov–Krasovskii functional (LKF) including new triple integral terms, free-weighting matrices, partly unknown transition probabilities and combining both the convex combination technique and reciprocal convex technique, a delay-dependent exponential stability criteria is obtained in terms of linear matrix inequalities (LMIs). The information about the lower bound of the discrete time-varying delay is fully used in the LKF. Furthermore, the desired sampled-data synchronization controllers can be solved in terms of the solution to LMIs. Finally, numerical examples are provided to demonstrate the feasibility of the proposed estimation schemes from its gain matrices.

1. Introduction

During the past few years, there have been increasing research interests in analyzing the dynamic behaviors of neural networks due to their extensive applications such as combinatorial optimization, adaptive control, signal processing, pattern recognition, image processing and association [1–3]. In electronic implementation of neural networks, time delays are frequently predictable in the process of information storage and transmission. It is well believed that the inherent time delays may cause oscillation and instability in many dynamical networks. In general, the time delays can be usually categorized as constant delays, time-varying delays, and distributed delays. Recently, LMI techniques have been successfully used to deal with various stability problems for neural networks with time delays (e.g., [4–6]). Therefore, the stability analysis of delayed neural networks had increased research interests in recent years.

As it is well known, Markov jump system is a special class of hybrid systems, which is specified by two components, the first component refers to the mode, which is described by a continuous-time finite-state Markovian process and second one refers to the state which is represented by a system of differential equations. The applications of the Markovian jump systems can be found in network control systems, manufacturing systems, economic systems, modeling production system, communication systems and so on. Stability analysis results on the Markovian jump neural networks can be found in [7–12]. Although, previous literatures usually assumed that the information on transition probabilities in the Markovian switching process is completely known. Some extended results are concerned with the uncertain transition probabilities [13]. But, practically in most cases transition probabilities on Markovian jump systems and networks are not exactly known. The DC motor in position control servomechanisms [14] for example, the changes of load or the inertia in different servomechanisms are vague and random, so it is hard to obtain all the elements in the expected transition probability matrix. The same problem may arise in other practical systems such as the networked control systems [15]. In these cases, the results developed for Markovian jump systems with completely known transition probabilities are not applicable. Therefore, it is significant and necessary to study more general jump systems with partially unknown transition probabilities. In [16–18], Zhang et al. investigated the problems of stability, stabilization and H_∞ filtering for a class of Markovian jump linear systems with partly unknown transition probabilities. In addition, the problems of stability and synchronization for a class of Markovian jump neural networks with partly unknown transition probabilities are discussed in [19]. Recently, stabilization of

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Markovian jump systems partially unknown transition probabilities via fuzzy control and $\mathcal{L}_2-\mathcal{L}_\infty$ for neutral Markovian switching systems with partially unknown transition probabilities are discussed in [20,21].

Really, chaotic neural networks as special complex networks can exhibit some complicated dynamics and are discussed in [22]. A typical characteristic of chaotic systems is their sensitive dependence on initial conditions. It means that it is generally difficult to achieve synchronization between chaotic systems. A large number of results have been presented on time delayed chaotic neural networks that have been widely used in various areas, such as secret communication, cryptography, pattern recognition, associative memory and combinatorial optimization. In [23], the authors have studied stabilization and synchronization control of Markovian jumping neural networks with mode-dependent mixed time delays subject to quantization and packet dropout. The adaptive synchronization for stochastic neural networks of neutral-type with mixed time-delays has been discussed in [24].

On the other hand, sampled-data control system has been studied extensively over the past few decades. In sampled data control method, the control signal is kept constant during the sampling period and is allowed to change only at the sampling instant. Also, in sampled data control systems, choosing proper sampling interval is more important for designing suitable controllers. In the past few decades, many of the researchers have discussed problems with constant sampling. Subsequently, researchers have also focused on time-varying sampling due to its applications in several practical systems. Hu et al. [25] studied the stability problem of digital feedback control systems with time-varying sampling periods. The authors in [26] have discussed the Integral control with variable sampling. Stochastic stability analysis for networked control systems by using time-varying sampling periods has been reported in [27]. Furthermore, the sampled-data systems have attracted great attention and the essential results have been proposed in [28–31]. The problem of exponential synchronization for neural networks with mixed delays using sampled-data feedback control has been discussed in [32]. In [33], the robust sampled-data $\mathcal{H}_\infty$ control problem has been investigated for active vehicle suspension systems. The authors in [34] dealt with the problem of sampled-data state estimation for delayed neural networks with Markovian jumping parameters. After that, some of the authors have discussed the sampled-data synchronization of various neural networks with time delays, see for example [35,36]. Obviously, it is meaningful and very interesting to study the problem of master–slave synchronization for neural networks with discrete delays using sampled-data control method for achieving less conservative delay-dependent conditions with a less number of decision variables to ensure that the master systems synchronize with the slave systems. To the best of authors′ knowledge, no related results have been established for the exponential synchronization of Markovian jump neural networks with discrete delays and partially unknown transition probabilities using stochastic sampling by incorporating with a convex combination technique.

Inspired by the above works, in this paper, we derive the criteria to exponential synchronization for Markovian jumping neural networks with unknown transition probabilities and stochastic sampled-data control using the Lyapunov stability theory. Also, the control gain matrices of the feedback controllers have been derived in terms of LMIs which can easily solved by any one of the LMI solvers [50]. With time dependent Lyapunov functional, convex combination technique and reciprocal convex technique, new stability criteria have been derived for the error dynamical systems by using sampled-data control with stochastic sampling. The main contribution of this paper is that the proposed results ensure the mean square exponential stability of the error system by using the newly designed delayed feedback controller in the slave system with unknown transition probabilities and in the system parameters. Two numerical simulations are finally given to show the effectiveness of the theoretical results.

Notations: Throughout this paper, $\mathbb{R}^n$ and $\mathbb{R}^{n\times n}$ denote the n-dimensional Euclidean space and the set of all $n\times n$ real matrices respectively. I denotes the identity matrix with compatible dimensions. $diag(\cdots)$ denotes a block diagonal matrix. The superscript T denotes the transposition and the notation $X\geq Y$ (similarly, $X>Y$), where X and Y are symmetric matrices, means that $X-Y$ is positive semi-definite (similarly, positive definite). Let $(\Omega,\mathfrak{F},\mathcal{P})$ be a complete probability space with a natural filtration $\{\mathfrak{F}_t\}_{t\geq 0}$. Also, let $d>0$ and $\mathcal{C}([-d,0];\mathbb{R}^n)$ denote the family of continuously differentiable functions ϕ from $[-d,0]$ to $\mathbb{R}^n$ with the uniform norm $\|\varphi\| = \max_{-d\leq\theta\leq 0}|\varphi(\theta)|$. Denote by $\mathcal{C}^2_{\mathfrak{F}_0}([-d,0];\mathbb{R}^n)$ the family of bounded $\mathfrak{F}_0-$measurable, $\mathcal{C}([-d,0];\mathbb{R}^n)-$valued stochastic variables $\xi=\{\xi(\theta): -d\leq\theta\leq 0\}$ such that $\int_{-d}^{0}\mathbf{E}||\xi(\theta)|^2\,ds<\infty$. $Pr\{\alpha\}$ means the occurrence probability of the event α. $\mathbf{E}\{x\}$ and $\mathbf{E}\{x|y\}$, respectively, mean the expectation of the stochastic variable x and the expectation of the stochastic variable x conditional on the stochastic variable y. $0_{m\times n}$ denotes the $m\times n$ zero matrix. $\Pi(i,j)$ denotes the ith row, jth column element (or block matrix) of matrix Π. The notation $*$ always denotes the symmetric block in one symmetric matrix.

2. Problem formulation and preliminaries

Consider the delayed neural networks with Markovian jumping parameters described by

$$\dot{x}(t) = -C(r(t))x(t)+A(r(t))g(x(t))+B(r(t))g(x(t-d(t)))+J(t), \tag{1}$$

where $x(t)=[x_1(t),x_2(t),\ldots,x_n(t)]^T\in\mathbb{R}^n$ is neuron state vector. The diagonal matrix $C(r(t))=diag(c_1(r(t)),\ c_2(r(t)),\ldots,c_n(r(t)))$ has positive entries $c_i(r(t))>0$ $(i=1,2,\ldots,n)$. The matrices $A(r(t))=(a_{ij}(r(t)))_{n\times n}$ and $B(r(t))=(b_{ij}(r(t)))_{n\times n}$ are the interconnection matrices representing the weight coefficients of the neurons. $J(t)$ is a constant input vector. $g(x(t))=[g_1(x_1(t)),\ g_2(x_2(t)),\ldots,g_n(x_n(t))]^T\in\mathbb{R}^n$ denotes the neuron activation function. Time varying delay satisfies $d_1\leq d(t)\leq d_2$, $\dot{d}(t)\leq\mu$, where $d_2>d_1>0$, μ are real constants. Let $\{r(t),t\geq 0\}$ be a right-continuous Markov chain on a complete probability space $(\Omega,\mathcal{F},\mathcal{P})$ taking values in a finite state space $\mathcal{S}=\{1,2,\ldots,N\}$ with generator $\Gamma=(q_{ij})_{N\times N}$ given by

$$P\{r(t+\Delta t)=j|r(t)=i\} = \begin{cases} q_{ij}\Delta t+o(\Delta t), & i\neq j,\\ 1+q_{ii}\Delta t+o(\Delta t), & i=j,\end{cases}$$

where $\Delta t>0$ and $\lim_{\Delta t\to 0}o(\Delta t)/\Delta t=0$, $q_{ij}\geq 0$ is the transition rate from i to j and if $i\neq j$, $q_{ii}=-\sum_{j=1,\ j\neq i}^{N}q_{ij}$.

Since the transition probability depends on the transition rates for the continuous-time Markov jump systems, the transition rates of the jumping process are considered to be partly accessible in this paper. For instance, the transition rate matrix for system (1) with N operation modes can be expressed as

$$\begin{bmatrix} q_{11} & ? & \ldots & ?\\ q_{21} & ? & \ldots & q_{2N}\\ \vdots & \vdots & \ddots & \vdots\\ ? & q_{N2} & \ldots & q_{NN}\end{bmatrix},$$

where "?" represents the unknown transition rate. Define, $\mathcal{S}=\mathcal{S}_1^i\cup\mathcal{S}_2^i$, $\forall i\in\mathcal{S}$, where $\mathcal{S}_1^i=\{j: q_{ij}$ is known$\}$ and $\mathcal{S}_2^i=\{j: q_{ij}$ is unknown$\}$.

Remark 1. When $\mathcal{S}_1^i=\mathcal{S}$, $\mathcal{S}_2^i=0$, it is reduced to the case where the transition probability rates of the Markovian jump process $\{r(t),t\geq 0\}$ are completely known. When $\mathcal{S}_1^i=0$, $\mathcal{S}_2^i=\mathcal{S}$, it means the transition probability rates of the Markovian jump process $\{r(t),t\geq 0\}$ are completely unknown. Here, we combine these two cases and consider a general form.