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A matheuristic for the asymmetric capacitated vehicle routing problem

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ABSTRACT

In this paper, we propose a novel matheuristic for the Asymmetric Capacitated Vehicle Routing Problem (ACVRP). This optimization-based approach combines some heuristic concepts with compact mixed-integer linear programming (MILP) formulations. Basically, the proposed matheuristic includes three sequential stages. First, the problem size is heuristically reduced by discarding unpromising arcs. Second, a starting feasible solution is derived. Finally, an optimization-based improvement procedure is invoked to iteratively generate near-optimal solutions. This latter procedure requires solving a sequence of two- or three-vehicle ACVRP reduced instances. A peculiar feature of the solution strategy is that all the three stages are solely based on solving compact MILP formulations using a commercial solver and it does not resort to any constructive heuristic nor metaheuristic. We describe the results of extensive computational experiments, that were carried out on a large set of benchmark instances with up to 200 nodes, and we provide empirical evidence that the proposed matheuristic often delivers high-quality solutions.

1. Introduction

In this paper, we deal with the NP-hard (in the strong sense) asymmetric capacitated vehicle routing problem (ACVRP). In the capacitated vehicle routing problem (CVRP), we are given a set of customers and a set of vehicles. Each customer is assigned a deterministic demand that cannot be split, all the vehicles are identical, based at a single depot and with a given capacity load. The problem requires finding a set of vehicle routes starting and ending at the depot, visiting and serving each customer only once, without exceeding each vehicle's capacity, while minimizing the total transportation costs (sum of the costs of the arcs belonging to the routes). In the ACVRP case, the cost matrix is asymmetric.

The CVRP/ACVRP is probably one of the most intensely investigated combinatorial optimization problems. For an overview of properties, solution approaches, and variants we refer to the books by Golden et al. [17] and by Toth and Vigo [23]. A review of the recent developments that had a major impact on the current state-of-the-art exact algorithms for the CVRP can be found both in [3] where mathematical formulations, relaxations and recent exact methods are analysed, and in [8] where combination of column and cut generation algorithms are presented. An overview of the recent heuristics and metaheuristics can be found in [20] and in [16] respectively.

Recently, matheuristics [4] appeared as a promising fourth alternative. An excellent survey on matheuristic for the vehicle routing problem has been proposed by Archetti et al. [1], where the matheuristics are divided into three classes:

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decomposition approaches, improvement heuristics and branch-and-price/column generation-based heuristics. For the decomposition method, an early contribution was given by Fisher and Jaikumar in [12], where a cluster first-route second heuristic was presented. In the cluster phase, the algorithm heuristically selects the so-called "seed customers" and assign to them the remaining customers by solving to optimality a generalized assignment problem, whereas in the route phase for each cluster a Travelling Salesman Problem (TSP) is solved. An improvement heuristic that uses a mathematical programming approach is proposed by De Franceschi et al. in [9], where, starting from a good initial solution known in the literature, the iterative procedure removes from the solution chains of customers, determines insertion points and solves a mathematical programming model in order to introduce the removed customers in the insertion points. A first application of column generation-based heuristic is reported by Foster and Ryan in [13], where a set partitioning formulation is presented together with a matheuristic in which petal routes are generated and a set partitioning formulation is solved. The afore-reported list of papers dealing with matheuristics is by no means exhaustive, for further details and in-depth analysis we refer to [1].

To the best of our knowledge, among the recent contributions that deal with the ACVRP, a heuristic algorithm has been proposed by Vigo in [26] and an ILP-based refinement heuristic has been presented by De Franceschi et al. [9]. Moreover exact approaches are reviewed by Toth and Vigo in [24] and a robust Branch-and-Cut-and-Price was proposed by Pessoa et al. in [7]. In [7], the authors modified the classical benchmark instances introduced by Fischetti et al. in [11] for the ACVRP by generating more instances with tighter capacities and with a larger number of vehicles.

Recently, effective hybrid approaches have been proposed by Subramanian et al. [22] and Vidal et al. [25]. In [22], the authors propose a hybrid algorithm for a class of VRPs with homogeneous fleet and in particular for the CVRP and the ACVRP. In the algorithm a sequence of Set Partitioning (SP) models whose columns correspond to routes found using an Iterated Local Search based metaheuristic, is solved, not necessarily to optimality, using a MILP solver. During the execution of the algorithm, the MILP solver and the metaheuristic interact with a reactive mechanism that dynamically controls the dimension of the SP models. In [25], a component-based heuristic design and a unified hybrid genetic search are proposed and the resulting metaheuristic turns out to be both efficient and applicable to a wide set of multi-attribute VRPs including the CVRP and the ACVRP.

In this paper, we present a novel matheuristic that can be framed into the class of decomposition approach in the classification in [1]. Indeed, we first apply a reduction on the number of variables of the problem in order to be able to find at least an initial feasible solution by solving to suboptimality a MILP model, and then, we improve the solution value identifying specific subproblems which are easier to be handled and solved independently using compact formulations. The solution of the original problem is, then, a combination of the solutions of the considered subproblems. The reported approach is applied to the ACVRP, however it results to be very flexible and it can be easily adapted to solve many variants of the VRP.

The whole matheuristic is uniquely based on solving MILP formulations and none of its stages benefits from any constructive heuristic or metaheuristic. Hence it could be considered a "pure" mathematical programming-based heuristic and because of this feature it does not have any presumption of outperforming the most competitive heuristics from a computational point of view. However, it aims at demonstrating that it can provide high-quality solutions that are competitive with state-of-the-art ACVRP heuristics within reasonable computation time.

The remainder of the paper is organized as follows. In Section 2, we introduce the notation and we describe two compact formulations that will be used as building blocks throughout the solution process. In Section 3, we present the different phases of the heuristic. Computational results on a large set of instances, with emphasis on the asymmetric benchmarks, are reported in Section 4, where we provide evidence that the presented approach delivers high-quality ACVRP solutions and near-optimal solutions for symmetric instances with up to 200 nodes. Finally, in Section 5, we provide some concluding remarks and outline future research directions that are worth investigating.

2. Compact mixed-integer programming models

The ACVRP can be formulated on a complete directed graph $G(V, A)$, where the set of the nodes V is the union of the set of all the customers V^* and of the singleton $\{0\}$ which represents the unique depot. Each arc $(i, j) \in A$ has an associated finite positive cost c_{ij} with $c_{ij} \neq c_{ji}$. The demand of node i is a positive integer indicated by d_i (with $d_0 = 0$ for the depot node). K is the set of the vehicles and $|K|$ its cardinality. All the vehicles are identical, therefore they have the same capacity load that is denoted by C.

Given node i, δ_i^+ and δ_i^- denote the forward and the backward star of i, respectively. Moreover, for sake of notation, we introduce the set $A^* := A \setminus \{(i, j) : i = 0 \text{ or } j = 0 \text{ or } i > j\}$ as the set of all the arcs (i, j) that are not incident to the depot and such that the index i is smaller than the index j.

Finally, in the sequel, $R_i \subset A$ stands for the route of vehicle i, $c(R_i)$ for its cost and $|R_i|$ for the number of arcs of R_i.

The mathematical models we use along the whole heuristic involve a polynomial number of variables and constraints in the size of the problem. We are interested in embedding compact formulations into the heuristic procedure with the following motivations: (i) they are flexible and can be easily extended to solve several VRP variants (e.g. VRP with heterogeneous vehicle fleet, multi-capacitated vehicle, etc.), (ii) they require relatively little coding efforts and thus can be more manageable for any user, and (iii) they are often amenable to solve small-sized instances optimally.