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Sports scheduling search space connectivity: A riffle shuffle driven approach

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ABSTRACT

The canonical method is widely used to build single round robin schedules for sports competitions. Certain properties of the canonical method may entrap local search procedures. In this paper, we study the connectivity of one of the most used neighborhood structures in local search heuristics for single round robin scheduling and characterize the conditions in which this entrapment happens. This characterization brings to light a relation between the connectivity of the analyzed neighborhood and the riffle shuffle, a method of shuffling playing cards.

1. Introduction

In a single round robin (SRR) tournament, an even number n of teams play against each other on $n-1$ rounds. Each team plays once every round, therefore each round consists of $n/2$ games. Every game involves two teams, say i and j. Hence, it is reasonable to associate each team with a vertex of a graph, every game involving i and j with an edge (i, j) and the schedule of an SRR tournament with an edge coloring of the corresponding graph. We show how graph theory provides an efficient tool for analyzing the solution space of sport scheduling problems.

Januario and Urrutia [8] conducted an investigation on the connectivity of the search space of existing neighborhood structures for SRR scheduling problems. Their analysis showed clear correlations between the method used to build initial schedules and the ability of local search procedures to escape from certain regions of the search space. Their study also showed that for several values of n the following phenomenon occurs: the two general neighborhood structures commonly used in local search based algorithms are not connected, meaning that not all the search space is reachable when using only those neighborhood structures. In particular, they showed that, for several values of n, when the most commonly used method in the literature (the canonical method, described in the next section) is used to build SRR schedules, the studied neighborhood structures were unable to reach schedules structurally different from (i.e., non-isomorphic to) the initial one.

For one of the neighborhood structures studied, called partial round swap (PRS), the phenomenon of non-connectivity could be explained by a classical result in graph theory related to perfect one-factorizations. The values of n for which this phenomenon occurs were determined to be equal to $p+1$, for every prime number $p > 2$.

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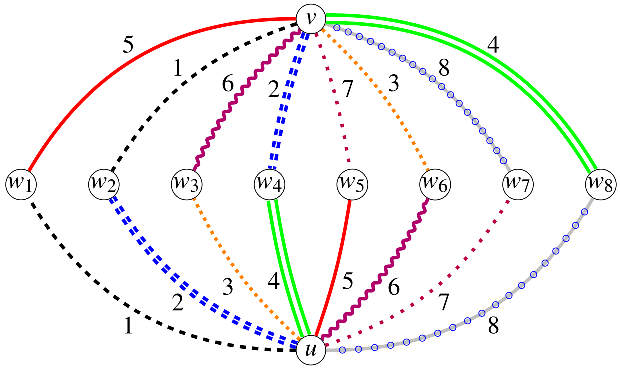

Fig. 1. A complete bipartite graph $B(X, W)$ isomorphic to $K_{2,8}$.

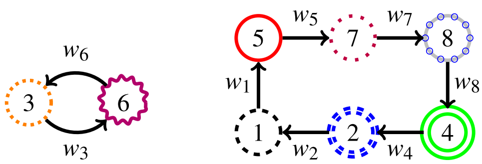

Fig. 2. The graph $\mathcal{G}$, generated from the bipartite graph in Fig. 2.

For some values of $n = p + 1$ (such as $n = 8$), the other neighborhood structure, called partial team swap (PTS), allowed the search to move to schedules non-isomorphic to the initial one. In order to understand and characterize the sequence of values of n for which the two neighborhood structures stay trapped within schedules that are isomorphic to the one initially constructed by the canonical method, Januario and Urrutia [8] empirically analyzed the sequence of numbers for which the phenomenon occurs and found a perfect correspondence with the sequence of numbers representing the size of a deck of playing cards in which the faro shuffle permutation has an $(n - 2)$-cycle [6,12].

Due to empirical evidences Januario and Urrutia [8] conjecture that PTS neighborhood is not able to escape from schedules isomorphic to the canonical one when the value of n is such that a faro shuffle permutation on a deck of n cards has an $(n - 2)$-cycle. In this paper, we prove their conjecture about the connectivity of PTS.

The rest of this paper is organized as follows. A description of the PTS neighborhood structure is given in Section 2. Section 3 introduces the canonical method, widely used to construct initial solutions for round robin sport scheduling problems. Next, a brief discussion about the faro shuffle is given in Section 4. An analysis of the PTS neighborhood structure is performed in Section 5 where we prove the conjecture in [8]. In the last section, we state some concluding remarks.

2. The structure of partial team swap

Different neighborhood structures have been proposed and used in local search procedures for round robin sport scheduling problems as in [2], [5] and [13]. Partial Team Swap (PTS) is frequently used in local search procedures to solve round robin sport scheduling problems.

In this section, PTS is described in graph theoretical terms. For more details about other neighborhood structures for round robin sport scheduling problems we refer to [8]. Let $n \geq 4$ be an even number and let $\{v_0, v_1, \ldots, v_{n-1}\} = V$ be the set of vertices of a complete graph K_n. We also consider a proper $(n - 1)$-coloring of the edges of K_n with colors $\{c_0, c_1, \ldots, c_{n-2}\}$. We will refer to the vertices and colors by their indexes. In that way, a given number t may refer either to a vertex v_t or to a color c_t. In each context, it should be clear whether we are referring to vertices or to colors.

A proper $(n-1)$-coloring of the edges of K_n is completely defined by the color $\mathcal{C}(i, j)$ of each edge (i, j) or by the function $\mathcal{O}(i, d)$ that returns the vertex linked to vertex i by an edge with color d.

Consider the complete bipartite graph $B(X, W)$, isomorphic to $K_{2,n-2}$, spanned by a set $X = \{u, v\}$, $u < v$, and a set $W = V \setminus \{u, v\} = \{w_1, w_2, \ldots, w_{n-2}\}$. Without loss of generality, we can assume that the edges of $B(X, W)$ are colored with colors $1, 2, \ldots, n - 2$, as can be seen in Fig. 1. These colors define a permutation π of $\{1, \ldots, n - 2\}$ by setting $\pi(\mathcal{C}(u, w_i)) = \mathcal{C}(v, w_i)$.

Let $S_{u,v}$ be a partition of W into subsets $S^1_{u,v}, \ldots, S^p_{u,v}$ such that for any $S^q_{u,v}$, with $1 \leq q \leq p$, the set of colors of edges (u, w), for $w \in S^q_{u,v}$, is the same as the set of colors of edges (v, w), for $w \in S^q_{u,v}$, and each $S^q_{u,v}$ is (inclusionwise) minimal. In graph theoretical terms, a move in PTS neighborhood structure consists in swapping the color assignment of edges (v, w_i) and (u, w_i) for each vertex $w_i \in S^q_{u,v}$, for a given q such that $1 \leq q \leq p$.

In order to construct the partition $S_{u,v}$, we build an auxiliary graph $\mathcal{G}$, see Fig. 2, as follows: each color $\{1, \ldots, n - 2\}$ is associated to a vertex in $\mathcal{G}$. Each vertex w_i in W corresponds to an oriented arc w_i of $\mathcal{G}$. The arc w_i is oriented from vertex k to vertex l if and only if (u, w_i) has color k and (v, w_i) has color l.

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