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Multi-objective linear programming games and applications in supply chain competition

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HIGHLIGHTS

- Multi-objective linear programming games are studied.
- Two kinds of solution methods are proposed for obtaining the equilibrium.
- The convergence results for the new algorithms are presented.
- The efficiency of the new algorithm is shown by an application to supply chain competitions.

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ABSTRACT

In this paper, we study a class of multi-player multi-objective game models with linear objectives. Based on the duality theory and Karush–Kuhn–Tucker (KKT) conditions, two approaches for solving the model are proposed, respectively. For the duality based approach, we show that solving Pareto equilibrium of the primal problem is equivalent to solving a multi-linear system. As to the KKT based approach, we prove that Pareto equilibrium can be achieved by solving a linear complementarity problem. As an application, we investigate supply chain competition problem using the underlying game model, which is further solved by these two approaches. Preliminary numerical test is conducted to demonstrate the efficiency of the presented approaches.

1. Introduction

As an extension to the traditional game with scalar-criterion, multi-objective game supposes the players (decision makers) in the game act by taking into account multiple criteria [1,2]. In literature, we can trace the study of multi-criteria games to the work of Blackwell [3] which studies the multi-criterion zero-sum games. Since then the study of multi-payoff game models becomes popular, for instance, see [4–7] and the references therein. Its popularity is largely due to the fact that most real-world problems in decision-making are with multiple decision criteria [8,9]. Consequently, it is suitable and appropriate to apply multi-objective models to tackle these problems. In this paper, we are interested to investigate a special class of multi-objective game models, namely, n-person multi-objective linear programming games in which each player has at least two linear objectives with linear constraints to be optimized simultaneously.

In multi-objective games, several competing objectives are considered for every player which makes it difficult or even impossible to simultaneously optimize all objectives [10]. To deal with this issue, people often resort to a weighted method where the decision maker measures the importance of each objective and then allots a set of nonnegative weights for objectives [11]. The decision maker now solve the resulted optimization problem with a weighted sum objective, provided that other decision makers also solve the resulted optimization problems with their own weighted sum objectives [4,5,12]. It is known that the resulted equilibrium by this way is called a weighted Nash equilibrium [13,14]. However, this method has some disadvantages particularly from the perspective of practical applications. In implementation, the players need to set the weights in advance while their actual values are not known in general. Then, ambiguity and uncertainty are inevitably involved when choosing the values of these weights [15,16]. In addition, as pointed out in literature, the player often needs to design some special methods to generate the weights [17,18], which implies that different weights can be obtained by the same player with different methods. Thus, this approach may affect the reliability of the obtained results.

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To overcome the ambiguity and uncertainty in choosing the weights, a robust weighted approach is proposed by assuming that the decision-maker cannot know the exact weights to its objectives but he knows that the weights may lie in a given set and then each player in the game reaches the optimal strategy by minimizing the maximum weighted sum objectives [7]. Although this method may overcome the shortcomings of the previous approach, the authors leave out the discussion about how to choose the weight set in practices. Moreover, the robust weighted approach is used to deal with the general multi-objective game. Multi-objective linear game is a special class of multi-objective game. Therefore, it is necessary and important to explore new approaches to solve multi-objective linear game models.

This paper aims at presenting new methods for solving Pareto equilibrium of multi-objective linear programming game models. This class of game models has a vast applications in supply chain management. For example, consider a multi-level supply chain network with uncertain costs in which the manufactures are all risk averse and compete with each other to meet the customers' demand. Each manufacturer needs to decide the production by considering the following two objectives, i.e., minimize the risk and maximize the profit simultaneously. In this supply chain multi-objective game, if the risk and profit functions are all linear functions, we can use the methods proposed in this paper to compute the corresponding Pareto equilibrium. Specifically, we propose two approaches for solving multi-objective games with linear objectives, which are based duality theory and KKT (Karush–Kuhn–Tucker) conditions, respectively. Durea et al. [19] introduce the notion of approximate KKT points for smooth, convex and nonsmooth, nonconvex vector optimization problems. A pair of primal and dual problems is introduced for multi objective linear programming problems under a fuzzy environment [20]. Using the duality method, the original multi-objective game can be cast as a scalar game with linear objective from which we establish a multi-linear system reformulation. As to the second approach, the primary game can be equivalently transformed into an equilibrium problem and it is known that there are well developed algorithms and software can effectively resolve the resulting problem. To demonstrate the advantage of proposed methods, a supply chain competition problem is examined using these two methods.

This paper is structured as follows. Section 2 discusses the optimality conditions and duality of multi-objective linear programming in addition to some necessary preliminaries for investigating multi-objective linear programming games. Two approaches based on the duality and KKT conditions are presented in Sections 3 and 4. Section 5 discusses an application in supply chain management. Section 6 concludes.

2. Preliminaries

2.1. Multi-objective linear programming

In this section, we recollect some results which will be used to develop the underlying duality method for solving Pareto equilibrium. Give the multi-objective programming as follows,

$$\min_{x\in X} F(x) \tag{2.1}$$

where $F : R^n \rightarrow R^m$ is a multi-objective function and $X \subset R^n$ is defined by a set of constraints. In multi-objective optimization, Pareto optimality (or weak Pareto optimality) is a fundamental concept used to determine optimal resource allocation strategy. Specifically, a Pareto optimum $x \in X$ (or a weak Pareto optimum) is defined as no strategy $y \in X$ exists such that the following condition holds

$$F(x) \preceq F(y) \ (or \ \ F(x) \prec F(y)).$$

Theorem 2.1 below presents the optimality conditions for problem (2.1) with linear objectives subject to linear constraints using the concept of Pareto optimality (or weak Pareto optimality), which can be proved using the methods proposed by Qu et al. [21] and Miettinen [22].

Theorem 2.1 (*Optimality Conditions*)**.** *For multi-objective optimization problem (2.1) with $F(x) := Cx$, $X := \{x|\ Gx \geq a,\ Hx = b\}$, $C \in R^{m\times n}$, $G \in R^{p\times n}$ and $H \in R^{q\times n}$, the following statements hold.*

(I) *A WPO (weak Pareto optimum) $\bar{x} \in R^n$ is equivalent to there exist $\lambda := (\lambda_1, \ldots, \lambda_m)^T \in \mathbb{R}^m_+$ with $\sum_{j\in I}\lambda_j = 1$ where $I := \{1, \ldots, m\}$, $\mu \in R^p_+$ and $\gamma \in \mathbb{R}^q$ such that*

$$C^T\lambda - G^T\mu + H^T\gamma = 0, \tag{2.2}$$

$$\mu^T(G\bar{x} - a) = 0,\ G\bar{x} \geq a,\ H\bar{x} = b. \tag{2.3}$$

(II) *If a point $\bar{x}$ is a PO (Pareto optimum) to (2.1), then there exist $\lambda := (\lambda_1, \ldots, \lambda_m)^T \in \mathbb{R}^m_+$ with $\sum_{j\in I}\lambda_j = 1$ and $(\mu, \gamma) \in \mathbb{R}^p_+ \times \mathbb{R}^q$ satisfying (2.2) and (2.3);*

(III) *If there exist $\bar{x} \in R^n$, $\lambda := (\lambda_1, \ldots, \lambda_m)^T \in \mathbb{R}^m_{++}$ with $\sum_{j\in I}\lambda_j = 1$ and $(\mu, \gamma) \in \mathbb{R}^p_+ \times \mathbb{R}^q$ satisfying (2.3) and (2.2), then $\bar{x}$ is a PO to (2.1).*

Theorem 2.2 (*Dual Theorem*)**.** *If $m = 1$ in (2.1) with $F(x) := Cx$, $X := \{x|\ Gx \geq a,\ Hx = b\}$, $C \in R^{m\times n}$, $G \in R^{p\times n}$ and $H \in R^{q\times n}$, the dual problem to the optimization problem (2.1) is as follows*

$$\begin{aligned} &\max_{y\in\mathbb{R}^p_+,\ z\in\mathbb{R}^q} \ a^Ty + b^Tz \\ &\text{s.t. } G^Ty + H^Tz = C^T. \end{aligned} \tag{2.4}$$

If (2.1) (or (2.4)) has a finite optimal solution, then so does (2.4) (or (2.1)), and these two respective optimal values are equal.

2.2. Multi-objective linear programming games

In this paper, we are interested to investigate multi-objective games with linear objectives as follows

$$MOLPG := \left(N, \{S_j\}_{j\in N}, \{F^j\}_{j\in N}\right),$$

where $N := \{1, \ldots, n\}$ denotes that there is a finite number of players,

$$S_j := \left\{x^j \in \mathbb{R}^{a_j} \,|\, A_jx \geq d_j,\ B_jx = b_j\right\}$$

is the set of actions for player j, $A_j \in \mathbb{R}^{p_j\times\sum_{j=1}^n a_j}$, $B_j \in \mathbb{R}^{q_j\times\sum_{j=1}^n a_j}$, $d_j \in \mathbb{R}^{p_j}$ and $b_j \in \mathbb{R}^{q_j}$, $\forall j \in N$. F^j is player j's vector-valued objective and is linear and is a mapping $S := \prod_{j\in N}S_j$ to R^{b_j}.

Throughout this paper, without loss of generality, it is supposed that there are multiple objectives for every player, i.e., $b_j \geq 2$. Let $F^j(x) := (f^j_1(x), \ldots, f^j_{b_j}(x))$ denote player j's multi-objective function in the game, where $x := (x^1, \ldots, x^n) \in S$ is the decision variable. Clearly, when $b_j = 1$, $\forall j \in N$, MOLPG reduces to linear scalar programming games. In literature, for the multi-objective game, each player adopts a strategy which is the (weak) Pareto optimal strategy and is a borrowed notion from multi-objective optimization problem [4–6].

Definition 2.3. Player j's strategy $x^j \in S_j$ is an optimal strategy, if given the other players' strategies $x^{-j} \in S_{-j} := \prod_{k\in N, j\neq k}S_k$, x^j is an optimum to the following problem,

$$\min_{x^j\in S_j} \ F^j(x), \tag{2.5}$$

i.e., $x^j \in S_j$ is a Pareto optimal strategy (or a weak Pareto optimal strategy) if x^j is a Pareto optimum (or a weak Pareto optimum) to (2.5).