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ORIGINAL ARTICLE

Multi-fuzzy Rough Sets based on Implicators and Continuous *t*-norms

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Abstract This paper extends the study of multi-fuzzy rough sets using an implicator and a continuous *t*-norm and thus introduces multi-fuzzy rough sets based on fuzzy logical connectives. In this constructive approach, a pair of lower and upper approximation operators determined by an implicator and a triangular norm is defined. The fundamental properties of these approximation operators are examined. Connections between multi-fuzzy relations and the newly constructed multi-fuzzy rough approximation operators are also established. The theory of multi-fuzzy rough sets is analysed using an operator oriented view in the later sections. The lower and upper approximation operators are characterized by axioms. Various axiom sets of lower and upper multi-fuzzy set theoretic operators guarantee the existence of different types of multi-fuzzy relations which produce the same operators.

Keywords Rough sets · Multi-fuzzy sets · Multi-fuzzy rough sets · Implicator · Triangular norm

1. Introduction

Rough set theory, introduced by Z. Pawlak [23] in 1982, has been described as an extension of set theory and a new mathematical tool to deal with vagueness and uncertainty. It can be effectively used to analyse information that is inaccurate, inconsistent and incomplete.

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The main idea of rough set theory is based on the indiscernibility relation that every object is associated with a certain amount of information, and the object can only be expressed with the aid of some obtained information. The basic operators in rough set theory are lower and upper approximations. The rough set method is to make a more identifiable data model based on the assumption of reducing the accuracy in data presentation.

Over the years, several improvements were made in rough set theory as well as in theory of hybrid structures involving rough sets. In the Pawlak's rough set model [23, 24], an equivalence relation is a primitive notion. But, later many authors have generalized the concept of approximation operators using non-equivalence binary relations such as Boolean algebra [20], T-similarity relations [6, 21], weak fuzzy partitions [5, 17], etc. Even though rough set theory is different from other generalizations of sets, such as fuzzy sets [33] and multi-sets [7], it is complementary to these theories. Because of its high complementary nature with other soft technologies, we can integrate the advantages of rough set theory with other theories. Fuzzification of rough sets is one such example [3, 12]. Dubois and Prade in [12, 13] proposed the concepts of fuzzy rough sets and rough fuzzy sets. The lower and upper approximations of fuzzy sets with respect to a crisp approximation space and a fuzzy approximation space result in a rough fuzzy set and fuzzy rough set, respectively. Fuzzy rough sets have been used to solve practical problems such as data mining [22], approximate reasoning [25], mining stock price [29] and so on. To overcome the shortcomings of traditional fuzzy rough set theory, Sun et.al. [8] proposed the interval-valued fuzzy rough set theory by combining the interval-valued fuzzy set theory with rough set theory. The theory of interval type-2 rough fuzzy sets and interval type-2 fuzzy rough sets [35, 36], intuitionistic fuzzy rough sets [30, 31], hesitant fuzzy rough sets [11], soft rough sets, rough soft sets [2, 34] are some examples of hybrid structures that combine the advantages of existing techniques. It is to be noted that studies have been made in applying rough set theory to algebraic structures [4, 10, 18].

Theory of multi-fuzzy sets [26, 27] is a generalization of theories of fuzzy sets, L-fuzzy sets, and intuitionistic fuzzy sets. Characterization problems like colour characterization of colour images, taste recognition of food items, decision-making problems with multi-aspects, etc. cannot be completely characterized by a single membership function while they can be characterized by multi-membership functions of suitable multi-fuzzy sets. The multi-fuzzy set theory has practical relevance in image processing [28].

The authors have studied multi-fuzzy relation based approximation operators in their earlier work [15]. The concept of rough multi-fuzzy sets and multi-fuzzy rough sets are introduced in [15] as a new tool to deal with the incomplete information in a multi-dimensional aspect. The aim of this paper is to introduce multi-fuzzy rough approximation operators in a more general way using a continuous triangular norm, $\mathcal{T}$ and an implicator, $\mathcal{I}$. The Section 2 reviews some basic concepts related to the work. A pair of $\mathcal{T}$-upper and $\mathcal{I}$-lower multi-fuzzy approximation operators is defined based on an arbitrary multi-fuzzy relation in Section 3. The properties of $\mathcal{T}$-upper multi-fuzzy rough approximation operators and $\mathcal{I}$-lower multi-fuzzy rough approximation operators are then examined in Sections 4 and 6, respectively. Sections 5 and 7 dis-