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An improved algorithm for computing a shortest watchman route for lines

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ABSTRACT

We give a new algorithm to compute a shortest watchman route for a set $\mathcal{L}$ of n non-parallel lines in the plane. A watchman route for $\mathcal{L}$ is a closed curve contained in the union of the lines of $\mathcal{L}$ such that every line of $\mathcal{L}$ is visited by the route. We show that the lines visited by the shortest watchman route can be specified in order, and then present an $O(n^6)$ time algorithm using dynamic programming technique. Our result significantly improves upon the previous $O(n^8)$ time bound.

1. Introduction

Shortest paths are of fundamental importance in computational geometry, robotics and autonomous navigation. The *watchman route problem*, introduced by Chin and Ntafos [1], is to find a shortest route (closed curve) in a given simple polygon P such that every point of P is visible from at least one point of the route. This problem is quite interesting, because it deals with both the visibility and metric information [6].

The first polynomial-time ($O(n^4)$) solution was given by Tan et al. [8,9] for the watchman route problem where a starting point on the boundary of P is given. This result has recently been improved to $O(n^3)$ [3,10]. The watchman route problem without giving any starting point can be solved in $O(n^4)$ time [7,10]. The solution to the watchman route problem has also been used to compute the shortest traveling salesman routes for n non-parallel lines in the plane [5]. The watchman route and related problems are usually characterized by the feature that the order of geometric objects to be visited is given or can be computed [3,6,10].

Recently, Dumitrescu et al. [4] considered the following variant: Given a set $\mathcal{L}$ of n non-parallel lines in the plane, one wants to find a shortest watchman route for $\mathcal{L}$ such that the route is contained in the union of the lines of $\mathcal{L}$ and it visits all lines (see Fig. 1). For this *watchman route problem for lines*, they gave an $O(n^8)$ time algorithm using dynamic programming technique, which is mainly based on an ordering of the vertices of the solution route [4]. Since its complexity is high, whether a faster algorithm can be given is left as an open problem. Note also that the watchman route problem for n line segments is NP-hard [4].

In this paper, we present an improved solution to the watchman route problem for lines. By investigating the order of lines in which the given lines are visited by a shortest watchman route, we develop a new dynamic programming algorithm with running time $O(n^6)$. Our result gives a significant improvement upon the previous $O(n^8)$ time solution.

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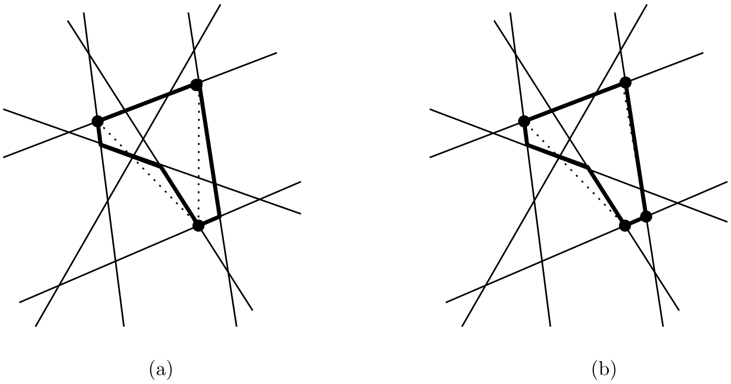

Fig. 1. Two solutions to MCH consist of three vertices (a) and four vertices (b) respectively; both yielding the same solution to WRL (drawn in bold).

2. Main result

For a set $\mathcal{L}$ of n non-parallel lines in the plane, denote by $\mathcal{A}(L)$ the arrangement of $\mathcal{L}$. Since there exist two non-parallel lines in $\mathcal{L}$, the arrangement $\mathcal{A}(L)$ is clearly connected. We also consider $\mathcal{A}(L)$ as a planar graph whose vertices are the vertices of the arrangement $\mathcal{A}(L)$ and whose edges connect successive vertices on the lines in $\mathcal{L}$. Denote by $\mathcal{A}_w(\mathcal{L})$ the weighted graph of $\mathcal{A}(L)$, where the weight of an edge is the Euclidean distance between its two vertices. For two different vertices x, y of $\mathcal{A}(L)$, represent by $\pi(x, y)$ a shortest path connecting x and y in $\mathcal{A}_w(\mathcal{L})$; the length of $\pi(x, y)$ is denoted by $|\pi(x, y)|$.

The watchman route problem for lines has been solved by relating it to the computation of the minimum convex hull defined on $\mathcal{A}(L)$ [4].

The watchman route problem for lines in the plane (WRL): Given a set $\mathcal{L}$ of non-parallel lines in the plane, compute a shortest watchman route for $\mathcal{L}$ in the graph $\mathcal{A}_w(\mathcal{L})$.

The minimum convex hull problem (MCH): Given a set $\mathcal{L}$ of non-parallel lines in the plane, compute a minimum length cyclic sequence $(v_1, \dots, v_k, v_1)$ of vertices of $\mathcal{A}_w(\mathcal{L})$ in convex position, such that every line in $\mathcal{L}$ intersects the convex polygon $(v_1, v_2 \dots, v_k)$, where the length of $(v_1, \dots, v_k, v_1)$ is defined as $\sum_{i=1}^{k} |\pi(v_i, v_{i+1})|$ with $v_{k+1} = v_1$. For simplicity, we also refer to the convex polygon $(v_1, \dots, v_k)$ as the solution to MCH.

It has been known that a solution to MCH is actually a solution to WRL [4]. There may be multiple solutions to MCH of the same length. For the instance given Fig. 1, two solutions to MCH are shown, and the same solution to WRL produced by them is drawn in bold.

Lemma 1. *([4]) For a set $\mathcal{L}$ of non-parallel lines in the plane, a solution to MCH yields a solution to WRL.*

The important observation made in this paper is that the lines visited by any solution to MCH can be specified in order. Assume that L is a non-vertical line of $\mathcal{L}$. Let L^+ (L^-) be the half-plane bounded by L and above (below) the line L. Assume that u is a vertex on L in $\mathcal{A}(L)$, and there are no lines that are parallel to and below L^+ (above L^-), i.e., there exists a watchman route in L^+ (L^-). Denote by $MCH(u^+)$ ($MCH(u^-)$) the minimum convex hull problem restricted to L^+ (L^-) and with the starting point u (i.e., the resulting convex hull must have vertex u). Let $R_{opt}(u^+)$ ($R_{opt}(u^-)$) be a solution to $MCH(u^+)$ ($MCH(u^-)$).

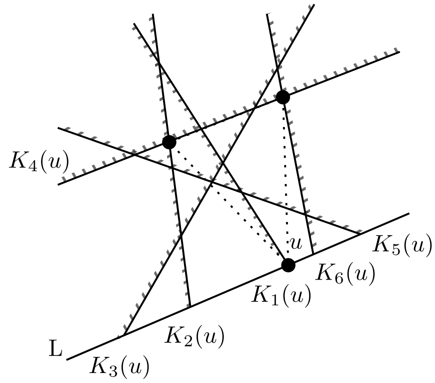

Fig. 2. An ordering of $K_1(u), K_2(u), \dots, K_{n-1}(u)$ on L.

In the following, we shall simply denote $MCH(u^+)$ by $MCH(u)$ and $R_{opt}(u^+)$ by $R_{opt}(u)$. For a line K, we shall denote by $K(u)$ the half-plane that is bounded by the portion of K inside L^+ and does *not* contain u, see Fig. 2. Then, $R_{opt}(u)$ visits $K(u)$ in one of the following ways.

1. It makes a *crossing contact* with $K(u)$, if R_{opt} contains points in either half-plane bounded by K. For the instance given in Fig. 2, R_{opt} makes crossing contacts with $K_1(u)$, $K_3(u)$ and $K_5(u)$.
2. In the case that $R_{opt}(u)$ does not go across $K(u)$, it comes into K at a vertex of $\mathcal{A}(L)$, and then, either goes away from that vertex immediately or walks a while along K and goes away from another vertex on K. The former is called a *reflection contact* with $K(u)$, and the latter is called a *tangential contact*, which can be considered as a degenerated reflection. In Fig. 2, $R_{opt}(u)$ makes reflection contacts with $K_2(u)$ and $K_6(u)$ and a tangential contact with $K_4(u)$.