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High-accuracy adaptive modeling of the energy distribution of a meniscus-shaped cell culture in a Petri dish

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ABSTRACT

Cylindrical Petri dishes embedded in a rectangular waveguide and exposed to a polarized electromagnetic wave are often used to grow cell cultures. To guarantee the success of these cultures, it is necessary to enforce that the specific absorption rate distribution is sufficiently high and uniform over the Petri dish. Accurate numerical simulations are needed to design such systems. These simulations constitute a challenge due to the strong discontinuity of electromagnetic material properties involved, the relative low field value within the dish cultures compared with the rest of the domain, and the presence of the meniscus shape developed at the liquid boundary. The latter greatly increases the level of complexity of the model in terms of geometry and intensity of the gradients/singularities of the field solution. In here, we employ a three-dimensional (3D) *hp*-adaptive finite element method using isoparametric elements to obtain highly accurate simulations. We analyze the impact of the geometrical modeling of the meniscus shape cell culture in the *hp*-adaptivity. Numerical results showing the error convergence history indicate the numerical difficulties arisen due to the presence of a meniscus-shaped object. At the same time, the resulting energy distribution shows that to consider such meniscus shape is essential to guarantee the success of the cell culture from the biological point of view.

1. Introduction

A Petri dish is a cylindrical plate often used for cell cultures. To induce and control the growth of these cell cultures, they may be exposed to various electromagnetic (EM) fields. One common scenario is to place the Petri dish into a rectangular waveguide that is illuminated with a polarized wave radiating at a particular frequency, [1,18,17].

In order to ensure the proper growth of the cell cultures, it is necessary to guarantee the high and uniform distribution of the EM energy (typically measured in terms of SAR – Specific Absorption Rate–) within the Petri dish [1,20]. Some authors state that in addition to control the SAR, one also needs to impose some additional conditions, e.g., polarization, on the distribution of the full electromagnetic fields to secure the proper evolution of the cell culture (see [22] and references therein). Furthermore, the meniscus shape developed at the interface of the liquid with the dish provides a complex shape, whose geometry is typically expressed as a mathematical formula involving exponential and hyperbolic functions.

Suitable numerical simulation methods for these scenarios need to have the following features. First, they should be able to handle three-dimensional geometries, including the Petri dish shape, and the meniscus shape. Second, they should be able to efficiently deal with the discontinuous material properties at the air-liquid interface. Third, they should be flexible enough to enable simulation of all possible scenarios, including various geometries, polarizations, and frequencies. Finally, and more importantly, since the observed electromagnetic fields produced by modifications on the design system are often small but nonetheless important, the simulation software should be highly accurate for all considered models. Moreover, it should provide an error estimation in order to minimize uncertainty and guarantee the correctness of the solution for each model.

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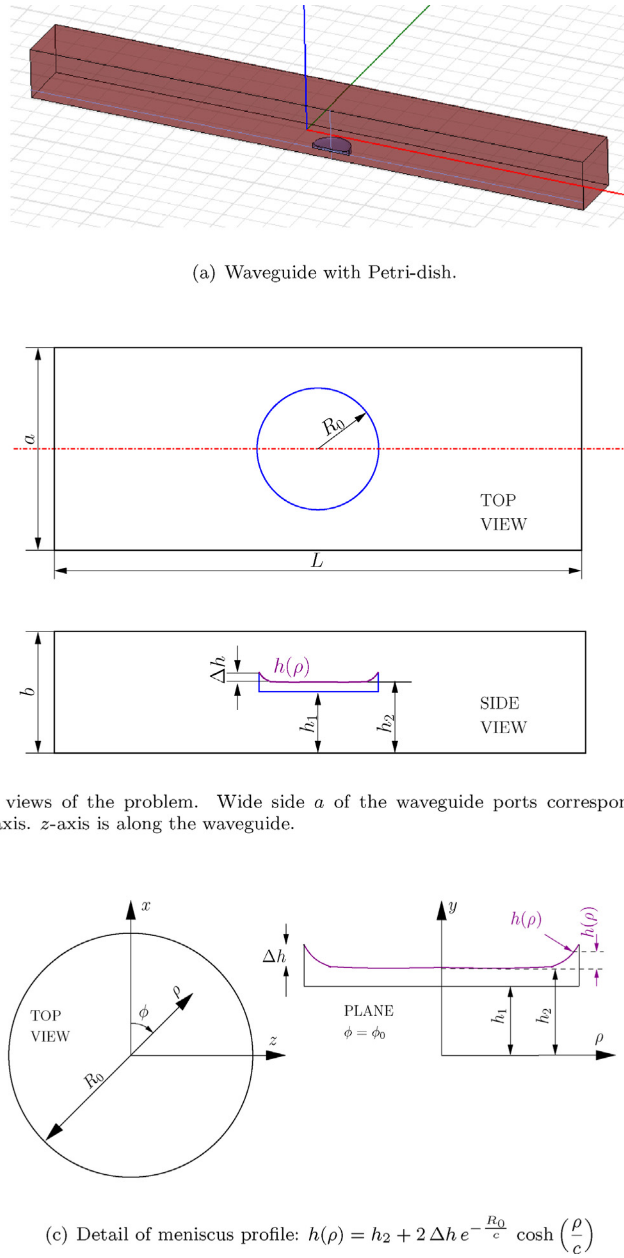

(a) Waveguide with Petri-dish.

(b) Top and side views of the problem. Wide side a of the waveguide ports correspond to x-axis. Side b corresponds to y-axis. z-axis is along the waveguide.

(c) Detail of meniscus profile: $h(\rho) = h_2 + 2\,\Delta h\, e^{-\frac{R_0}{c}} \cosh\left(\frac{\rho}{c}\right)$

Fig. 1. Problem geometry. Dimensions expressed in millimeters: $a = 95$, $b = 45$, $R_0 = 16.9$, $L = 300$, $h_1 = 27.5$, $h_2 = 30.5$, $\Delta h = 2.51$.

Several numerical methods have been employed for Petri dish simulations exposed to EM fields in different configurations, e.g., [3,20,21,16,1,22,23]. Approaches based on differential formulations are mainly used because of their flexibility to deal with complex geometrical and material configurations. Among them, the most common numerical technique is finite differences (FD); typically, in time domain.

SAR data is obtained by averaging the energy distribution within a small cube shaped volume (known as voxel), which is the natural choice in FD grids. The presence of non-Cartesian geometries, boundary layers, and field singularities encountered on the resulting field solution together with internal resonances, requires the use of tiny voxels. Unfortunately, the use of small voxel sides of the order of one tenth (or even one hundredth) of a wavelength may not be enough in some cases to have confidence on the results. In those situations, the voxels close to the solid/liquid interface are skipped from the SAR distribution assessments, as it is reported in [22].

In here, we propose to employ a highly accurate method that works under all the above scenarios and provides an error estimation that guarantees the correctness of the solution. It is based on a Finite Element Method (FEM) that utilizes “adapted” meshes to both the geometry of the problem domain and its solution. A sequence of adapted meshes is generated in an automatic fashion by refining a given mesh in certain areas of the domain. Simultaneous h and p refinements, i.e., local variations of the element size h and the polynomial order of approximation p throughout the mesh are supported (the so called *hp*-adaptivity [5,6]). Preliminary results of the 3D implementation of the *hp*-adaptivity proposed by the authors [9] applied to the Petri dish problem were presented in [10], where the cell cultures were modeled as a circular dielectric, i.e., the meniscus shape was not included in the geometry. In this paper, the meniscus shape is included in the computational model. The main focus of this work is to analyze: (a) the numerical effect of the *geometrical refinements* in the *hp*-adaptivity due to the presence of a meniscus-shaped object, and (b) the differences obtained in the