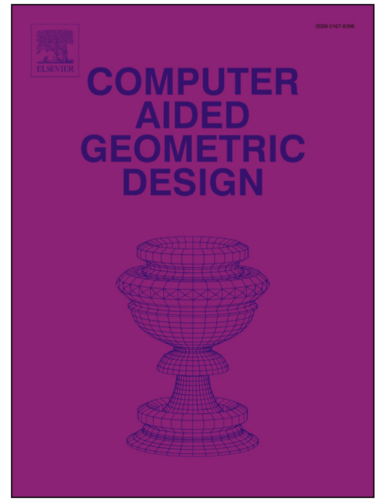
Accepted Manuscript



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PII: S0167-8396(18)30037-2
DOI: https://doi.org/10.1016/j.cagd.2018.03.023
Reference: COMAID 1680

To appear in: *Computer Aided Geometric Design*

Elliptic Grid Generation Techniques in the Framework of Isogeometric Analysis Applications

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Abstract

The generation of an analysis-suitable computational grid from a description of no more than its boundaries is a common problem in numerical analysis. Most classical meshing techniques for finite-volume, finite-difference or finite-element applications such as the *Advancing Front Method* [19], *Delaunay Triangulation* [20] and elliptic or hyperbolic meshing schemes [21] operate with linear or multi-linear but straight-sided elements for the generation of structured and unstructured meshes, respectively, whereas the generation of high-quality curved meshes is still considered a major challenge. A recent development is the introduction of *Isogeometric Analysis* (IgA) [12], which can be considered as a natural high-order generalisation of the finite-element method. A description of the geometry $\bar{\Omega}$ is accomplished via a mapping operator $\mathbf{x} : \hat{\Omega} \to \Omega$ that maps the unit hypercube in $\mathbb{R}^n$ onto an approximation Ω of $\bar{\Omega}$ utilizing a linear combination of higher-order spline functions. The numerical simulation is then carried out in the computational domain $\hat{\Omega}$ via a *'pull back'* using the mapping operator $\mathbf{x}$. The advantage is that the flexibility of higher-order spline-functions usually allows for an accurate description of $\bar{\Omega}$ with much fewer elements which can significantly reduce the computational effort required for this step compared to traditional low-order methods. Furthermore, an analytical description of the geometry can be turned back into a traditional (structured or unstructured) grid by performing a large number of function evaluations in $\mathbf{x}$. This can, for instance, be utilized for local refinement without the need for remeshing.

A potential drawback of curved instead of linear elements is that the meshing techniques required for the creation of folding-free mappings tend to be more sophisticated and that it is a less trivial task to verify that the resulting mapping is indeed bijective.

For the purpose of creating folding-free mappings utilizing spline-functions, we will present an algorithm adopting the principles of *Elliptic Grid Generation* (EGG) whose basic principles have been adapted to meet the needs of IgA. In $\mathbb{R}^2$, EGG has particularly appealing properties since bijectivity of the resulting mapping is guaranteed as long as the numerical accuracy to the computational approach of the mapping is sufficient. We will present an algorithm that is capable of generating folding-free mappings from a large number of geometry contours, including complicated geometries from industrial applications with extreme aspect ratios. This is accomplished by combining EGG with automatized reparameterization techniques and a sophisticated numerical approach for solving resulting governing (nonlinear) equations. The algorithm is equipped with the means to verify the bijectivity of the resulting mapping and with automatized defect-correction methods in case a violation of bijectivity is detected.

Furthermore, we will present possible strategies for the generation of folding-free mappings for certain types of volumetric geometries by combining EGG with transfinite-interpolation as well as a number of other applications such as time-dependent settings. All applications are provided with example geometries.

Contents