Computers & Graphics 74 (2018) 191–201

Contents lists available at ScienceDirect

Computers & Graphics

journal homepage: www.elsevier.com/locate/cag

Special Section on SMI 2018

Peeling the longest: A simple generalized curve reconstruction algorithm

Amal Dev Parakkat [a], Subhasree Methirumangalath [b], Ramanathan Muthuganapathy [a,*]

[a] Advanced Geometric Computing Lab, Indian Institute of Technology Madras, India
[b] Department of Computer Science & Engineering, National Institute of Technology Calicut, India

ARTICLE INFO

Article history:
Received 28 April 2018
Revised 14 May 2018
Accepted 15 May 2018
Available online 23 May 2018

Keywords:
Curve reconstruction
Delaunay triangulation
Noise simplification
Self-intersection

ABSTRACT

Given a planar point set sampled from a curve, the curve reconstruction problem computes a polygonal approximation of the curve. In this paper, we propose a Delaunay triangulation-based algorithm for curve reconstruction, which removes the longest edge of each triangle to result in a graph. Further, each vertex of the graph is checked for a degree constraint to compute simple closed/open curves. Assuming ϵ-sampling, we provide theoretical guarantee which ensures that a simple closed/open curve is a piecewise linear approximation of the original curve. Input point sets with outliers are handled as part of the algorithm, without pre-processing. We also propose strategies to identify the presence of noise and simplify a noisy point set, identify self-intersections and enhance our algorithm to reconstruct such point sets. Perhaps, this is the first algorithm to identify the presence of noise in a point set. Our algorithm is able to detect closed/open curves, disconnected components, multiple holes and sharp corners. The algorithm is simple to implement, independent of the type of input, non-feature specific and hence it is a generalized one. We have performed extensive comparative studies to demonstrate that our method is comparable or better than other existing methods. Limitations of our approach have also been discussed.

1. Introduction

Curve reconstruction of a given set of points S, sampled from a curve C, computes a polygonal (piecewise linear) approximation of the curve. The curve C can be an open or a closed curve with self-intersections, disconnected components, multiple holes and sharp corners, where a hole (inner boundary) is considered as a convex/non-convex simple polygon which is enclosed within a boundary. The input and output of the problem are as shown in Fig. 1(a) and (b), respectively. Even though the reconstruction problem, in general, has a rich literature over the last three decades, it is still an active and challenging problem due to its ill-posed nature [1]. Nevertheless, it has various applications in the fields of computational geometry, computer vision, computer graphics, image processing and pattern recognition.

Edelsbrunner proposed a Delaunay triangulation-based parametric method to produce α-shape [2], which characterizes the shape of a point set. Even though it was not designed for curve reconstruction, subsequently its 3D version [3] was shown to be applicable for reconstruction. $\mathcal{A}$-shape [4] is computed from a combination of Delaunay triangulation and Voronoi diagram. Veltkamp proposed a method for reconstruction which results in γ-graph [5]. Duckham et al. [6] proposed a parametric method for reconstruction from the Delaunay triangulation to produce characteristic shape (χ-shape). Locally Density Adaptive α-complex [7] is a Delaunay triangulation-based, locally adaptive method for curve reconstruction. Simple shape [8] computes the curve, based on the distance, angle and certain other criteria. Concepts of Delaunay disks are used to compute r-regular shape [9]. Crust algorithms [10–12] use a combination of Delaunay triangulation and Voronoi diagram to produce closed/open curves. In Crust, a dense sampling based on medial axis transform was introduced by Amenta et al. [10], which is widely used to ensure theoretical guarantee of a reconstructed curve. Reconstruction using nearest neighbor graph with theoretical guarantee is presented in nn-crust [11]. In power crust [12], a subset of Voronoi vertices known as poles is used to build a power diagram, which divides the plane into interior and exterior cells. Noise filtering of a given point set and introducing new points, followed by pruning and reconstruction using nn-crust is proposed by Cheng et al. [13]. Mehra et al. [14] proposed a visibility operator on the convex hull of a noisy point set and in turn used the visibility information to perform both curve and surface reconstructions. Feiszli et al. [15]

* Corresponding author.
E-mail addresses: adp.upasana@gmail.com (A.D. Parakkat), subhasree.rajiv@gmail.com (S. Methirumangalath), mraman@iitm.ac.in (R. Muthuganapathy).

https://doi.org/10.1016/j.cag.2018.05.015
0097-8493/

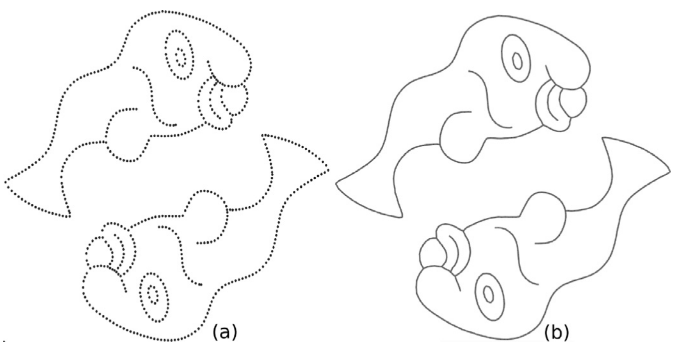

Fig. 1. (a) Input point set. (b) Our output with closed/open curve, disconnected components, self-intersections, multiple holes and sharp corners.

introduced a non-parametric denoising strategy and reconstructing a curve preserving sharp corners. However, the three curve reconstruction algorithms mentioned above [13–15] do not reconstruct open curves, disconnected components, curves with self intersections and they are not designed for handling outliers. Lee [16] proposed a reconstruction method based on moving least squares concept, specially designed for noisy point sets to compute curves without self-intersections. Shape hull [17] removes the edges of a Delaunay triangulation based on the position of circumcenter of triangles to construct a simple closed divergent curve. Another Delaunay triangulation-based method is ec-shape, which use empty circle approach for outer boundary detection [18] and hole detection [19]. Non-divergent curves are also reconstructed by ec-shape, but not open curves. Water-distribution-model (wdm) crust [20] is based on Voronoi diagram and handles outliers. Crawl [21] reconstructs closed/open curves with disconnected components and multiple holes, however it does not handle noisy point set. Optimal transport cost method proposed by de Goes et al. [22] is a greedy method to minimize the increase in the transport cost, which is designed for noisy point sets. Wang et al. [23] proposed a quad-tree method with smoothening concept to reconstruct a curve from a noisy point set with outliers. There are algorithms such as Fidelity vs. Simplicity [24], which perform a piecewise smooth reconstruction of a given sketch.

Most among the above methods are designed only for a simple closed curve reconstruction [6,17,18] whereas a few of them [21–23] reconstruct both open and closed curves. To the best of our knowledge, only few of them [10,11,20] provide theoretical guarantee.

Apart from reconstructing open and closed curves, few algorithms are designed to handle other challenges of reconstruction problem such as (i) the input can be noisy or/and with outliers (ii) the original curve can have disconnected components, self-intersections, multiple holes and sharp corners. Even though a few of the reconstruction algorithms [18,20,21] detect some of the features mentioned above, they are not designed for handling noise. There are algorithms specially designed for handling noise [13–15], but they do not reconstruct open curves, disconnected components, curves with self intersections and are not designed for handling outliers. Lee [16] designed a reconstruction algorithm for noisy point sets for both closed and open curves, however, it is not able to detect self-intersections. de Goes et al. [22] and Wang et al. [23] detect self-intersections on a noisy point set, however performance of Wang et al. [23] degrades if the input is without noise. Also, how to detect the presence of noise remains to be a challenging open problem. Moreover, many algorithms have multiple parameters, hence it is very tedious to synchronize and tune.

Table 1 summarizes the comparison of the existing methods, where *Y* or *N* refers to YES or NO based on whether the corresponding method handles the following: Open Curve (OC), Disconnected Components (DC), Self-Intersections (SI), Noisy Input (NI), Input with Outliers (IO), Non-Feature-Specific (NFS) and Number of Parameters (NP).

Table 1
Summary of comparison of existing methods, where *Y* or *N* refers to YES or NO based on whether the corresponding method handles the following: Open Curve (OC), Disconnected Components (DC), Self-Intersections (SI), Noisy Input (NI), Input with Outliers (IO), Non-Feature-Specific (NFS) and Number of Parameters (NP). *Crust and nn-crust are not specifically designed for reconstructing self-intersections, noisy inputs and outliers, hence, the reconstructed output may or may not handle these. @ One parameter for simplifying the noisy point set and another one for identifying the presence of self-intersections.

Summary of comparison							
Method	OC	DC	SI	NI	IO	NFS	NP
α-shape [2]	Y	Y	Y	N	N	Y	1
χ-shape [6]	N	N	N	N	N	Y	1
Simple shape [8]	N	N	N	N	N	Y	3
Crust [10]	Y	Y	Y*	N*	Y*	Y	0
nn-crust [11]	Y	Y	Y*	N*	Y*	N	0
Cheng et al. [13]	N	N	N	Y	N	N	3
Mehra et al. [14]	N	N	N	Y	N	N	2
Feiszli et al. [15]	N	N	N	Y	N	N	0
Lee [16]	Y	Y	N	Y	Y	Y	0
Shape hull [17]	N	N	N	N	N	N	0
ec-shape [18,19]	N	N	N	N	N	Y	1
wdm-crust [20]	N	N	N	N	Y	Y	0
Crawl [21]	Y	Y	N	N	Y	Y	0
de Goes et al. [22]	Y	Y	Y	Y	Y	N	2
Wang et al. [23]	Y	Y	Y	Y	Y	N	4
Our method	Y	Y	Y	Y	Y	Y	2@

In general, it is challenging to develop a generalized algorithm which handles all the features - closed/ open curves, disconnected components, self-intersections, multiple holes and sharp corners as well as handle noise and outliers.

1.1. Our contributions

We have developed an algorithm which reconstructs simple closed/open curves with theoretical guarantee. Point sets with outliers are also handled by this algorithm without pre-processing. Further, we have designed strategies to identify the presence of noise and self-intersections. Our algorithm is enhanced to simplify a noisy point set and perform curve reconstruction. It is also extended to reconstruct curves with self-intersections. The algorithm is simple to implement too. Hence, our algorithm is a generalized one as it is not designed for a particular input case or feature-specific, but tuned to handle different input cases and detects various features.

Our major contributions are as follows:

- Non-feature specific algorithm for reconstruction of closed/open curves.
- Novel ‘flower structure’ to identify the presence of noise (perhaps for the first time), using Delaunay triangulation.
- Strategy to identify and restore self-intersections in the reconstructed curve.

2. Algorithm

Let *DT* denote the Delaunay triangulation [25] of an input point set *S*. An edge between a pair of points (vertices) in *DT* is denoted by *e*. It has been proved that, if *S* is obtained by ϵ-sampling [10] from a simple closed curve, a set of connected subgraphs of *DT* provides a piecewise linear approximation of *S* [10]. Hence, we use this fact to develop our reconstruction algorithm, based on Delaunay triangulation.