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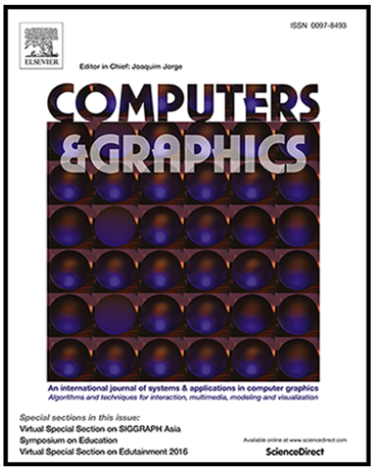

A Heat Flow Based Relaxation Scheme for n Dimensional Discrete Hyper Surfaces

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A Heat Flow Based Relaxation Scheme for n Dimensional Discrete Hyper Surfaces

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ABSTRACT

We consider the problem of relaxing a discrete $(n-1)$ dimensional hyper surface defining the boundary between two adjacent n dimensional regions in a discrete segmentation. This problem often occurs in computer graphics and vision, where objects are represented by discrete entities such as pixel/voxel grids or polygonal/polyhedral meshes, and the resulting boundaries often expose a typical jagged behavior. We propose a relaxation scheme that replaces the original boundary with a smoother version of it, defined as the level set of a continuous function. The problem has already been considered in recent years, but current methods are specifically designed to relax curves on triangulated discrete 2-manifolds embedded in $\mathbb{R}^3$, and do not clearly scale to multiple discrete representations or to higher dimensions. Our biggest contribution is a smoothing operator entirely based on three canonical differential operators: namely the Laplacian, gradient and divergence. These operators are ubiquitous in applied mathematics, are available for a variety of discretization choices, and exist in any dimension. To the best of the author's knowledge, this is the first intrinsically dimension-independent method, and can be used to relax curves on 2-manifolds, surfaces in $\mathbb{R}^3$, or even hyper-surfaces in $\mathbb{R}^n$. We demonstrate our method on a variety of discrete entities, spanning from triangular, quadrilateral and polygonal surfaces, to solid tetrahedral meshes.

1. Introduction

Labeling (or *segmenting*) an object is a fundamental operation in computer graphics and vision, widely used in applications such as analysis of medical images, object recognition and detection.

The majority of segmentation algorithms work on discrete domains, such as regular grids [1], polygonal [2, 3, 4, 5] and polyhedral [6, 7] meshes. A common approach consists in assigning to each element of the domain a value (or *label*). Elements sharing the same label belong to the same region, whereas elements with different labels belong to separate regions. As a consequence, boundaries between adjacent regions are only intrinsically defined, and amount to the union of the interfaces between adjacent elements.

Given a n dimensional object and a labeling defined on it, the boundary between a pair of adjacent regions is a $(n-1)$ dimensional hyper surface. As a concrete example one may consider a binary partition of a discrete two dimensional surface (e.g. a triangle mesh): the boundary between the two regions is the chain of edges having polygons with opposite labels at its sides. The same goes for three-dimensional objects (e.g. a tetrahedral mesh): the boundary is the set of faces having polyhedra with opposite labels at its two sides. Indeed, the boundary is one dimensional if the object is two dimensional, and is two dimensional if the object is three dimensional.

Depending on the quality of the discretization, both in terms of number, regularity, and shape of each element in the domain, the boundaries between adjacent regions can be geometrically poor, showing a typical jagged behaviour (Figure 1). In this

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