Graphical Models 97 (2018) 80–88

Contents lists available at ScienceDirect

Graphical Models

journal homepage: www.elsevier.com/locate/gmod

Stress-oriented structural optimization for frame structures

Shuangming Chai[a], Baiyu Chen[a], Mengyu Ji[a], Zhouwang Yang[a], Manfred Lau[b], Xiao-Ming Fu[a], Ligang Liu*[,a]

[a] School of Mathematical Sciences, University of Science and Technology of China, Hefei 230026, China
[b] School of Computing and Communications, Lancaster University, Lancaster LA1 4WA, UK

ARTICLE INFO

Keywords:
3D printing
Fabrication
Stress analysis
Optimization
Topology simplification

ABSTRACT

To fabricate a virtual shape into the real world, the physical strength of the shape is an important consideration. We introduce a framework to consider both the strength and complexity of 3D frame structures. The key to the framework is a stress-oriented analysis and a semi-continuous condition in the shape representation that can both strengthen and simplify a structure at the same time. We formulate a novel semi-continuous optimization and present an elegant method to solve this optimization. We also extend our framework to general solid shapes by considering them as skeletal structures with non-uniform beams. We demonstrate our approach with applications such as topology simplification and structural strengthening.

1. Introduction

We have witnessed many developments in the area of computational fabrication in recent years [1–4]. As 3D printers become increasingly common and affordable, there is a great need for tools that consider the physical properties of virtual objects. When bringing virtual objects into the real world through 3D printing, the strength of an object is one such important consideration. We present a framework to analyze both the strength and complexity of 3D frame structures, where a structure consists of a set of beams. The motivations for focusing on frame structures are that these structures are common in architectural models, they can also represent general 3D shapes, and they can be 3D printed as a real-world frame structure to represent the virtual shape without an excessive amount of printing material.

There has been some recent work exploring the idea of structural analysis of 3D printed shapes [5–7]. There also exists work that analyze structural parts and reduce the cost of 3D printing by building a skin-frame structure [8] or a honeycomb-cells structure [9]. However, previous methods optimize the structure problem iteratively between two parts: a geometry optimization and a topology optimization. In this paper, we introduce a single stress-oriented framework to analyze the strength and topology complexity of 3D frame structures simultaneously. A key contribution different from previous work is that we have a problem formulation and a semi-continuous condition in our shape representation. This condition can remove structurally redundant elements to reduce the overall shape complexity without sacrificing its structural strength.

We formulate our problem to optimize scalar parameters of a frame structure such that it is 3D printable with high strength, while taking into account the volume, semi-continuous, symmetric, and sparsity constraints. These constraints are quite intuitive, as they limit the size and complexity of the output structure while maintaining its aesthetics. The idea is to strengthen weak parts while maintaining the overall volume of the shape, and optionally changing the topology and keeping the shape symmetry. In particular, the semi-continuous constraint is key to our formulation, as it includes a choice between lower and upper bounds and a value of zero for each parameter in the shape representation. An element within a shape with a parameter value of zero will disappear. Our formulation of this condition allows us to explore the tradeoffs between strength and complexity in frame structures. The user can also control the tradeoff to choose among structures with various simplified topologies and high strength.

We use stress as a measure of strength of an object. We consider the frame structure as a set of beam elements and compute the stress of each element. Our stress-oriented structural optimization then minimizes the maximal stress of all elements. The semi-continuous condition in our shape representation requires a non-trivial solution to this problem. Hence we formulate a novel semi-continuous optimization and present an elegant method, the alternation direction method of multipliers (ADMM) algorithm, to solve it. Based on our framework on frame structures, we extend it to general tetrahedral meshes. By considering a tetrahedral mesh as a skeletal structure, the formulation and

* Corresponding author.
E-mail addresses: kfckfckf@mail.ustc.edu.cn (S. Chai), chenby@mail.ustc.edu.cn (B. Chen), jimengyu@mail.ustc.edu.cn (M. Ji), yangzw@ustc.edu.cn (Z. Yang), manfred.lau@gmail.com (M. Lau), fuxm@ustc.edu.cn (X.-M. Fu), lgliu@ustc.edu.cn (L. Liu).

https://doi.org/10.1016/j.gmod.2018.04.002
Received 19 December 2017; Received in revised form 7 April 2018; Accepted 30 April 2018
Available online 04 May 2018

optimization method can be easily applied to a skeletal structure.

We demonstrate our framework with various 3D models of frame structures. We show the applications of the strengthening of weak parts and topology simplification while maintaining structural symmetry. Our results highlight the main concepts of the stress-oriented structural optimization.

The contributions of our work are: (i) A stress-oriented framework to analyze both the strength and structural complexity of 3D frame structures with a semi-continuous condition; (ii) a semi-continuous optimization to minimize the maximal stress of a structure and an elegant method to solve this optimization; (iii) an extension of our framework to tetrahedral meshes by constructing skeletal structures from them and considering them as frame structures; and (iv) applications of our framework to structural strengthening and topology simplification. Section 3 describes the stress analysis framework and the representation of an input shape as a frame structure. Section 4 describes our problem formulation including the objective function, the semi-continuous condition, and various constraints in our stress-oriented optimization. Section 5 describes a reformulation of the original problem formulation into a semi-continuous optimization such that it can be solved with the ADMM algorithm. Section 6 describes the extension of our framework to tetrahedral meshes.

2. Related work

2.1. Structural analysis for fabrication

With the rapid development of techniques for 3D printing, many researchers have recently studied geometric processing problems for the purpose of fabrication. These fabrication-aware methods are typically led by a stress analysis that uses the finite element method. Stava et al. [5] introduce a method that analyzes the stress of a model and deforms it by hollowing, thickening and strut insertion. Zhou et al. [6] present a method to search for the worst-case stress under forces from all possible directions. Langlois et al. [10] use a stochastic finite element method to compute failure probabilities which can analyze the static soundness of one object. Zhou et al. [11] present a direct shape optimization method which penalizes geometric deviation while bounding the stress under specific external loads. Chen et al. [12] propose a finite element discretization scheme to use a reduced basis for fast stress analysis. Chen et al. [13] analyze elastic deformation caused by gravity to solve the inverse problem of computing a shape that when fabricated deforms naturally to a target shape. Prévost et al. [3] explore the problem of deforming shapes to make them physically balance. Among this area of work, we contribute a stress-oriented problem formulation for automatically strengthening a frame structure while simplifying its structural complexity.

2.2. Structural simplification for fabrication

There has been work on problems which aim to simplify the complexity of the structure of 3D shapes for fabrication. Many methods are based on decomposing a 3D shape into smaller pieces and then assembling them to form the original shape or a resemblance of it. Luo et al. [14] suggest a method to 3D print large objects by first segmenting an object into smaller parts and then assembling the parts to form the larger shape. Hildebrand et al. [15] create parts from a 3D shape that can then be fabricated and assembled in an optimal direction. Interlocked planar [16,17] or solid pieces [18–20] can be used to form a shape that resembles the original. Zimmer et al. [21] approximate surfaces by building physical structures with the Zometool construction system. Vanek et al. [22] present a method to divide a mesh into parts which are then efficiently packed into space for 3D printing. Instead of decomposing a shape into smaller pieces, we simplify a frame structure by possibly removing elements from it. We contribute a semi-continuous optimization for this purpose.

2.3. Special structure design

Motivated by existing architectural structures, some types of special 3D printed structures have been explored. Some structures, like skin-frame structure [8] or honeycomb-cells structure [9], are designed to reduce the cost of 3D printing via stress analysis. These methods are used for constructing the interior supporting structure of a solid object and these structures are cost-effective and are stable with high strength. Some structures are designed as the support structure necessary for 3D printing. The reduction of support structure can save printing time and material [4,23]. A bridge structure [4] can reduce the cost and meet stability conditions. Yang et al. [24] design a support-free structure to fabricate a balanced object without interior supports. Recently inspired by materials in nature, the idea of microstructures [25–28] become popular, and these are composed of tileable and printable small scale assemblies made of one or several materials. A framework is proposed [29] to generate statically sound and materially efficient frame structures with different types of cross sections. In this paper, we focus on strengthening and simplifying frame structures consisting of beams. We also extend our framework such that we can convert general meshes to skeletal representations for our analysis.

3. Preliminaries

This section describes the stress analysis and the representation of an input 3D shape of our algorithm as a frame structure. The stress computation described here is used in our optimization in Section 4.

3.1. Stress computation

In continuum mechanics, stress is a physical quantity that expresses the internal forces that neighboring particles of a continuous material exert on each other [30,31]. The strength of a material is measured in force per unit area, which depends on its capacity to withstand axial stress, shear stress, bending, and torsion. A static elastic object satisfies the following equilibrium equation:

$$\begin{cases} -\operatorname{div}\boldsymbol{\sigma}(\mathbf{u}) &= \mathbf{f}, \quad \text{in } \Omega, \\ \mathbf{u} &= \mathbf{0}, \quad \text{on } \Gamma_H, \\ \boldsymbol{\sigma}(\mathbf{u})\cdot\mathbf{n} &= \mathbf{g}, \quad \text{on } \Gamma_N, \end{cases} \tag{1}$$

where $\mathbf{u}$ is the displacement, $\boldsymbol{\sigma}$ is the stress tensor, $\mathbf{f}$ is the body force, and $\mathbf{g}$ is the surface force. This differential equation is defined in the region of an object Ω, Γ_H and Γ_N are two open subsets of the boundary of Ω, such that $\partial\Omega = \overline{\Gamma}_H \cup \overline{\Gamma}_N$ and $\Gamma_H \cap \Gamma_N = \varnothing$. We take the discretized form of this system, i.e. the linear equilibrium equation:

$$K\mathbf{u} = F, \tag{2}$$

where K is the stiffness matrix and F is the external loads including body forces and surface forces. Note that the displacement $\mathbf{u}$ is in the finite element space of piecewise linear continuous functions, which is different from the space in the continuous case (Eq. (1)).

3.2. Frame structure

As shown in Fig. 1, a frame structure consists of a set of beams and nodes where the beams are connected to each other at the nodes. In our framework, each beam is regarded as a simple cylindrical shape with a radius and a length. The beams defines the topology (i.e. the connectivity between nodes) of the frame structure. According to theory on frame structures [32,33], the stiffness matrix K in Eq. (2) can be computed for a frame structure, where K depends on the node positions and beam radii [8]. The forces in this equation are gravity or external loads we specify. We can then solve for the displacement and compute the stress for each beam in the frame structure. Although the stress varies in different parts of the beam, we only consider the largest stress for each