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Variable structure controllers for unstable processes

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ABSTRACT

A variable structure control (VSC) method for unstable industrial processes is proposed. The proposed control method is able to provide a highly satisfactory system performance and to tackle with robustness issues of the processes in the presence of uncertainties. An ITAE-based numerical tuning algorithm for acquiring optimal control parameters, and a direct auto-tuning mechanism for the proposed controller are also provided. The performance of the proposed VSC method is illustrated on some unstable process models including a continuous stirred tank reactor (CSTR), in order to show its effectiveness, validity and feasibility.

1. Introduction

Process control system designs are mostly based on PID controllers and empirical process models [1–5]. The empirical first-order plus time delay (FOPTD) models can describe dynamics of many processes appropriately for control design aims. Specifically such models are used for tuning PID controllers and stability analysis of the closed-loop processes [1]. The intuitiveness, simplicity and good performance features of the PID (mostly PI) controllers make them the most widely used control strategy today [6–9]. However, PID controllers, like other classical approaches, have robustness vulnerability and may pose performance challenges for unstable and uncertain processes.

In recent years, there has been a great interest in control designs for unstable processes (e.g. unstable FOPTD models) since it is well-known that the performance specifications obtained for a stable model cannot work for an unstable processes. Therefore, many methods have been developed for stabilizing unstable processes including the modified Ziegler–Nichols method [10], mirror mapping [11], truncated predictor feedback control [12], smith predictor based control [13], PID-based controllers [14–18], IMC method [19,20], optimization-based methods [21,22], synthesis method [23], sliding mode control [24,25], and the fuzzy-neural approach [26]. Most of the above methods have additional adjustable parameters for obtaining controller parameters, complex design procedures, and robustness issues.

The goal of this work is to develop a robust, simple and effective VSC method for unstable processes. Another aim is to provide a direct auto-tuning algorithm for the proposed VSC system without needing a secondary relay method. In the literature, there are very few VSC systems for unstable systems while different switching control strategies similar to gain scheduling approaches can be seen. Most of the given switched-systems have a switching strategy with some PID controllers or continuous change of controller parameters [27]. Some of the studies seen in the literature include: variable structure PID controllers [27–30], a variable parameters based PID controller [31,32], a controller switching between P, PD and PID structures [33,34], and a variable parameters based control [35]. In general, these studies utilize various switching logics to enhance the system performance under operational variations. Some studies have also been considered a specific variable structure system, i.e. sliding mode control methodology [36–39], for the process control systems [24,25,40–43]. In these studies, the integral sliding surface design was used for the reduced-order (FOPDT) models of processes, and some parameter tuning structures similar to empirical PID tuning algorithms were developed for process control systems. However, these methods require the measurement of the derivative of the process output, and thus, can result in poor control performances. For these reasons, this work aims to develop a VSC method having the robustness and good response features of the sliding mode control, and effectiveness and simplicity of PID controllers. Since process control systems often use empirical

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models, i.e. FOPDT models, robustness and setpoint performance of these controllers in the presence of parameter variations and disturbances become important in operations [1,44]. The proposed VSC approach with its direct auto-tuning function can solve all these issues by providing a good setpoint response and robustness.

The organization of the study is as follows: Section 2 introduces the proposed VSC system, some application examples of the controller are given in Section 3, and finally, the conclusion of this work is provided in Section 4.

2. The proposed variable structure controller

Some exothermic chemical reactors and biochemical reactors are operated at open-loop unstable steady-states [45]. For open-loop unstable systems, which are difficult to control due to the tight tuning requirements, an appropriate controller must first stabilize the system. In addition, model uncertainty, load disturbance, measurement noise, and set-point response must all be taken into account in a reasonable design method. A drawback of classical (e.g. PID) controllers is that they do not consider all these aspects in a balanced way [1]. More importantly, the robustness of controllers is an unavoidable problem in classical control methods. The robustness problem can be better addressed with a variable structure control system.

An open-loop unstable process can be modeled with [46,47]

$$\frac{Y(s)}{U(s)} = \frac{K}{\tau s - 1} e^{-Ls} \tag{1}$$

where K is the static gain, τ is the time constant and L is the delay. With the use of Taylor series approach, i.e. $e^{-Ls} \approx 1/(Ls+1)$, the unstable model (1) can be approximated to

$$\frac{Y(s)}{U(s)} \cong \frac{K}{(\tau s - 1)(Ls + 1)} \tag{2}$$

or in the differential equation form

$$\ddot{y} + \alpha_1 \dot{y} + \alpha_0 y = \beta u \tag{3}$$

where $\alpha_0 = 1/\tau L$, $\alpha_1 = (\tau - L)\alpha_0$ and $\beta = K\alpha_0$. Now, a VSC can be designed to stabilize error dynamics. Due to their inherently robustness against uncertainties, the VSC systems can fit well to such models with a suitable design. In these control methods, a switching surface is usually designed so that the VSC drives the error trajectories of a system onto this surface and keeps these trajectories on the surface for all subsequent times. For the system (3), a switching surface can be designed as

$$\sigma = \dot{e} + \lambda e \tag{4}$$

where $e = r - y$ with a reference signal $r(t)$, and $\lambda > 0$ is a constant. To bring error trajectories on this surface and keep them there, a suitable VSC can be designed by

$$\begin{aligned} u &= -k_0 y + k_p |\sigma|^{1/2} \mathrm{sat}(\sigma) + u_1 \\ \dot{u}_1 &= k_i |\sigma|^{1/2} \mathrm{sat}(\sigma) \end{aligned} \tag{5}$$

where k_0 and k_p are proportional gains, k_i is an integral gain, σ is the switching surface, and $\mathrm{sat}(\cdot)$ is the saturation function defined by

$$\mathrm{sat}(\sigma) = \begin{cases} \sigma/|\sigma|, & \text{if} \quad |\sigma| \geq 1 \\ \sigma, & \text{if} \quad |\sigma| < 1 \end{cases} \tag{6}$$

In (5), it is assumed that $|u_1| \leq \mu_3$ for some $\mu_3 > 0$. The block diagram of the proposed control system is illustrated in Fig. 1.

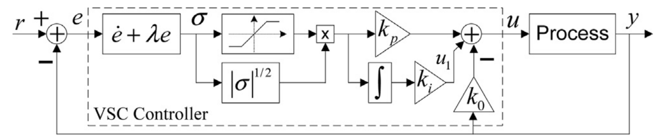

Fig. 1. Block diagram of the proposed VSC system.

By considering the model (3), switching surface (4), and controller (5), a stability analysis of the proposed VSC can be done as follows. First, if a positive definite Lyapunov function is defined by

$$V = \sigma^2/2 + u_1^2/2 \tag{7}$$

then its derivative must be negative definite for stability,

$$\dot{V} = \sigma\left(\ddot{r} + \lambda\dot{r} + (\alpha_1 - \lambda)\dot{y} + \alpha_0 y - \beta u\right) - \beta u_1 \sigma + u_1 \dot{u}_1 \tag{8}$$

where it is assumed that $\left|\ddot{r}\right| \leq \mu_2$, $\left|\dot{r}\right| \leq \mu_1$, $|y| \leq \mu_0$ for some positive numbers μ_0, μ_1 and μ_2. From (6), it is clear that the controller consists of outer and inner parts. For the outer part of the controller, i.e. for $|\sigma| \geq 1$,

$$\dot{V} = \sigma\left(\ddot{r} + \lambda\dot{r} + (\alpha_1 - \lambda)\dot{y} + (\beta k_0 - \alpha_0)y - \beta k_p |\sigma|^{1/2} sgn(\sigma)\right) + \varphi_0 \tag{9}$$

where the function φ_0 can be written as

$$\begin{aligned} \varphi_0 &= -\beta u_1 \sigma + k_i u_1 |\sigma|^{1/2} sgn(\sigma) \\ &= -\beta u_1 \sigma \left(1 - |\sigma|^{-1/2}\right) \\ &= -\varepsilon \beta u_1 \sigma \end{aligned} \tag{10}$$

where $0 \leq \varepsilon < 1$ and it is assumed that $k_i = \beta$ for simplicity. Substituting (10) into (9) results in

$$\begin{aligned} \dot{V} &= \sigma\left(\ddot{r} + \lambda\dot{r} + (\alpha_1 - \lambda)\dot{y} + (\beta k_0 - \alpha_0)y - \beta k_p |\sigma|^{1/2} sgn(\sigma)\right) + \varphi_0 \\ &\leq \left(\left|\ddot{r}\right| + \lambda\left|\dot{r}\right| + \varepsilon\beta|u_1| - \beta k_p |\sigma|^{1/2}\right)|\sigma| \\ &\leq \left(\mu_2 + \lambda\mu_1 + \varepsilon\beta\mu_3 - \beta k_p |\sigma|^{1/2}\right)|\sigma| \\ &= -\left(\beta k_p |\sigma|^{1/2} - \hat{\mu}\right)|\sigma| \end{aligned} \tag{11}$$

where $\hat{\mu} = \mu_2 + \lambda\mu_1 + \varepsilon\beta\mu_3$, $\lambda \geq \alpha_1$ and $k_0 = \alpha_0/\beta = 1/K$. Since $|\sigma| \geq 1$, if we choose $k_p > \hat{\mu}/\beta$, then $\dot{V} < 0$. Namely, whenever $|\sigma| \geq 1$, $\left|\sigma(t)\right|$ will strictly decrease until it reaches the set $|\sigma| < 1$ in finite time and remains inside the set subsequently. For the inner part of the controller, i.e. inside the set $|\sigma| < 1$, the Eq. (8) can similarly be written as

$$\dot{V} = \sigma(\ddot{r} + \lambda\dot{r} + (\alpha_1 - \lambda)\dot{y} + (\beta k_0 - \alpha_0)y - \beta k_p |\sigma|^{1/2}\sigma) + \varphi_1 \tag{12}$$

with

$$\begin{aligned} \varphi_1 &= -\beta u_1 \sigma + k_i u_1 |\sigma|^{1/2} \sigma \\ &= -\beta u_1 \sigma (1 - |\sigma|^{1/2}) \\ &= -\varepsilon_1 \beta u_1 \sigma \end{aligned} \tag{13}$$

where $0 < \varepsilon_1 < 1$ and again it is assumed that $k_i = \beta$ for simplicity. Finally,

$$\begin{aligned} \dot{V} &\leq \bar{\mu}\,|\sigma| - \beta k_p |\sigma|^{5/2} \\ &\leq -\left(1 - \theta\right)\beta k_p |\sigma|^{5/2} \end{aligned} \tag{14}$$

where $0 < \theta < 1$. The inequality (14) is satisfied for all

$$|\sigma| \geq \left(\frac{\bar{\mu}}{\theta\beta k_p}\right)^{2/3} \tag{15}$$

Hence, the trajectory reaches the ultimate bound set $\Sigma = \left\{|\sigma| < \left(\bar{\mu}/(\theta\beta k_p)\right)^{2/3},\ |\sigma| < 1\right\}$ in finite time. This means that the