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A weighted variable gain super-twisting observer for the estimation of kinetic rates in biological systems

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ABSTRACT

The knowledge of kinetic reaction rates is important for monitoring and controlling biotechnological processes. However, the lack of on-line sensors for this purpose and the inherent problems with numerical differentiation make observers indispensable. In this work, we propose the use of a weighted variable gain super-twisting observer (WVGSTO), applicable to a class of second-order nonlinear systems that include a measurable weight on the unmeasured variable and the possibility of bounding the perturbations with measurable functions. This estimation method is illustrated with an academic example and then applied to a fed-batch bioprocess.

1. Introduction

Biotechnological processes, including also the biodegradation used in the wastewater treatment, have become an important part of modern life, and its appropriate control, optimization and supervision is clearly a fundamental issue. The increasing incentive to develop Process Analytical Technologies (PAT) [1] has also boosted the implementation of on-line instrumentation, and in turn the interest in designing software sensors, which blend the information from a process model and from available on-line measurement signals to reconstruct other, non measured, variables.

These tasks are particularly challenging since the dynamical models of bioprocesses are highly uncertain and there is a lack of reliable and/or economical sensors for key variables. This explains the interest in the last decades for developing estimation strategies for the states and the (specific) reaction rates in bioprocess models [2].

An approach to deal with model variability and uncertainty is to consider known ranges for the parameter values within a simplified model. For example, interval observers [3,4] do not provide an estimate of the system trajectory but rather upper and lower bounds for this trajectory. The extended Kalman filter or other robust linear observers may deal with some degree of parameter uncertainty or signal noise, but they rely heavily on the model.

Asymptotic observers (AO) [2,5] are highly robust, since they are able to estimate the states of a bioreactor without the knowledge of the reaction rates. However, this requires the measurement of at least as many state variables as the number of reaction rates and usually the convergence of the observer cannot be assigned. The properties of AOs can be explained by using the theory of unknown input observers [6].

For the estimation of the (specific) reaction rates, considered as unknown inputs, high-gain observers (HGO) [7,8] have been successfully used [9–11]. A drawback of HGOs, and in general of any continuous observer, is that they are unable to estimate without error the reaction rates, even in the absence of measurement noise. This is due to the lack of knowledge of the velocity of variation of the reaction rate, and this uncertainty cannot be completely compensated by continuous observers. In order to reduce the estimation error the high gain of a HGO has to be increased, but this increases the sensitivity to measurement noise of the estimator [12].

An important feature of discontinuous observers, and in particular of higher order sliding-mode observers [13–15] is that they are able to estimate an unknown input *exactly and in finite time* despite the lack of knowledge of the rate of change of the signal. In particular, for the reaction rate estimation in bioreactors, the use of observers based on the super-twisting algorithm (STA), a second

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order sliding-mode algorithm, have been recently reported. In [16] a super-twisting observer (STO) is used to estimate the reaction rate in a chemical reactor to improve the control. The authors of [17–20] provide a modification of the STO to estimate the specific reaction rate in a bioreactor, which is multiplied by a measured signal (the biomass concentration). The proofs are based on previous results of [21,22] and a time scaling; they also provide experimental evidence.

In this paper we extend the results of [15,21–26] and provide a generalized super-twisting observer (WVGSTO) which is able to estimate an unmeasured state *exactly and in finite time*, when this state is multiplied by a known time varying signal which does not change sign. Using the WVGSTO it is possible not only to generalize and to extend, but also to unify, many results in the literature for the estimation of the reaction rates in bioprocesses, including the STO in [17–20] and the HGO in [9–11]. We show that in this case a quadratic Lyapunov function with *constant* matrix P allows us to assure the convergence of the observer in finite time and despite the perturbations using time-varying gains. We provide in the paper a study example to illustrate the application of the WVGSTO for the estimation of reaction rates and states in a particular bioprocess.

2. Problem statement

We consider the class of second order systems that are described by the (possibly multivalued or discontinuous) differential equations (to keep notation simple we do not indicate time dependence explicitly if it is obvious from the context):

$$\dot{x}_1 = f_1(x_1, u) + b(t, u, y)x_2 + \delta_1(t, x, u), \tag{1a}$$

$$\dot{x}_2 = f_2(x_1, x_2, u) + \delta_2(t, x, u, w), \tag{1b}$$

$$y = x_1, \tag{1c}$$

where $x_1 \in \mathbb{R}$, $x_2 \in \mathbb{R}$ are the states, $u \in \mathbb{R}^m$ is a known input, $w \in \mathbb{R}^r$ represents an unknown input and $y \in \mathbb{R}$ is the measured output; f_1 is a known continuous function and f_2 corresponds to a known possibly discontinuous or multivalued function. δ_1 and δ_2 represent uncertain terms. The measured variables are x_1 and the known input u. The signal $b(t, u, y)$ is a *known* positive function, and which is lower and upper bounded,[1] i.e.

$$0 \le b_m \le b(t, u, y) \le b_M. \tag{2}$$

It is assumed that system (1) has solutions in the sense of Filippov [27].

When the uncertainty $\delta_1(t, x, u) \equiv 0$ in (1) the observability map is

$$\mathcal{O}(x, u, w) = \begin{bmatrix} y \\ \dot{y} \end{bmatrix} = \begin{bmatrix} x_1 \\ f_1(x_1, u) + b(t, u, y)x_2 \end{bmatrix},$$

which is clearly globally invertible for every known and unknown input u and w. In the absence of unknown input w, system (1) (with $\delta_1(t, x, u) \equiv 0$ and $\delta_2(t, x, u, 0) \equiv 0$) is *uniformly observable for every input* [7,28]. When there is an unknown input w the system (with $\delta_1(t, x, u) \equiv 0$) is said to be *strongly observable* [6,29]. In both cases it is theoretically possible to determine the unmeasured state x_2 from the measurement of x_1. Note that if the uncertain term $\delta_1(t, x, u) \neq 0$, then observability is lost and it is impossible to determine exactly the state x_2.

Many second order systems are described by Eq. (1). For example, it can represent a mechanical system when $\delta_1(t, x, u) \equiv 0$ and $f_1(x_1, u) \equiv 0$, where x_1 corresponds to the (measured) position and x_2 is the velocity; u then represents a control force (or torque) and w corresponds to uncertain parameters or forces. If there exist Coulomb friction forces or in the presence of back-slash or hysteresis, the functions $f_2(x, u)$ and/or $\delta_2(t, x, u, w)$ are discontinuous or multivalued.

Many other systems, although not represented by (1) in original coordinates, can be brought to (1) by a (local or global) state diffeomorphism. In particular, it is well known that smooth systems (without uncertainties), which are uniformly observable for every input [7,28], can be transformed to the form (1).

The aim of this contribution is to propose an observer that is able to estimate the unmeasured state x_2 from the measurement of x_1. It is clear from the observability analysis that this will be possible in an exact manner only if the perturbation term $\delta_1(t, x, u) \equiv 0$ (we are only considering the case without measurement noise).

Technically, our main result is to extend the results presented in [15,21,22,26,25], which consider $b(t, u, y)$ in (1) to be constant to the case when this *weight* coefficient is time-varying but does not change its sign. We show that in this case a quadratic Lyapunov function with *constant* matrix P ensures convergence of the observer in finite time despite the perturbations using time-varying gains. Furthermore, we propose a practical approach to calculate these gains.

Since many existing observer algorithms can be used for this purpose, we will list the distinguishing properties of the proposed observer:

1. It is able to estimate exactly the state x_2 after a *finite time* and *robustly with respect to uncertainties/perturbations*, represented by $\delta_2(t, x, u, w)$ in (1), that are persistent. In order to achieve this feature it is necessary to introduce discontinuous functions in the injection terms of the observer. It is important to note that finite time convergence can also be achieved with continuous injection terms (though not locally Lipschitz in the neighborhood of the origin), but only in the absence of uncertainties/perturbations.
2. The proposed observer is able to converge in a finite time that is *independent of the initial condition* of the plant and of the observer. In order to achieve this property it is required to introduce not globally Lipschitz injection terms.
3. The observer is able to deal with a known function f_1 that is continuous but not necessarily Lipschitz (globally or locally). The function f_2 can be discontinuous, it does not have to be locally or globally Lipschitz in x_1 and it can grow linearly in x_2.
4. When a bounded uncertainty/perturbation δ_1 is present, the estimation error will be bounded. The same will be true in the presence of measurement noise.
5. All proofs are based on Lyapunov's method. The Lyapunov functions used here are of quadratic type, so that the mathematical machinery required is very similar to what is needed for linear systems.
6. The proposed method can be considered as a generalization and improvement of other observer design methods in the literature. In particular, it includes a linear observer with variable-gains and the high-gain observer [7–11].

3. Design method and properties of the proposed observer

In order to achieve the features for the observer listed before, in this section we propose a (discontinuous) observer, named *weighted variable-gain generalized super-twisting observer (WVGSTO)*, for system (1). We also describe how it is designed and discuss its properties.

[1] The case where $b(t, u, y)$ is always strictly negative can be treated similarly.