ARTICLE IN PRESS

Computers and Mathematics with Applications ▮ (▮▮▮▮) ▮▮▮–▮▮▮

Contents lists available at ScienceDirect

Computers and Mathematics with Applications

journal homepage: www.elsevier.com/locate/camwa

Pricing puttable convertible bonds with integral equation approaches

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ARTICLE INFO

Article history:
Received 30 March 2017
Received in revised form 2 December 2017
Accepted 4 January 2018
Available online xxxx

Keywords:
Puttable convertible bond
Fourier transform
Integral equation approach

ABSTRACT

American-style puttable convertible bonds are often priced with various numerical solutions because the predominant complexity arises from the determination of the two free boundaries together with the bond price. In this paper, two forms of integral equations are derived to price a puttable convertible bond on a single underlying asset. The first form is obtained under the Black–Scholes framework by using an incomplete Fourier transform. However, this integral equation formulation possesses a discontinuity along both free boundaries. An even worse problem is that this representation contains two first-order derivatives of the unknown exercise prices, which demands a higher smoothness of the interpolation functions used in the numerical solution procedure. Thus, a second integral equation formulation is developed based on the first form to overcome those problems. Numerical experiments are conducted to show several interesting properties of puttable convertible bonds.

1. Introduction

A convertible bond (CB) is one of the widely-used hybrid financial instruments. It gives the holder the right to convert a bond into a predetermined number of stocks at any time during the life of the bond, or to hold the bond until maturity to receive the principal payment. Such a conversion right gives the holder the possibility to gain a maximum benefit. But, this particular conversion feature has made the valuation problem more complicated because the optimal conversion boundary needs to be determined as part of the solution of the problem.

The theoretical framework for pricing CBs under the Black–Scholes model was initially proposed by Ingersoll [1] and Brennan & Schwartz [2]. They priced a convertible bond by using contingent claims, in which they took the firm value as the underlying variable. However, the model is not practical since the firm value is not observable in market. In 1986, McConnel & Schwartz [3] proposed a single-factor pricing model for a zero-coupon convertible bond, using the stock price as the underlying variable.

Since then, various approaches have been proposed to price convertible bonds. Analytical solutions are only available for CBs with very simple exercise clauses. For example, Nyborg [4] obtained a closed-form solution for a simple convertible bond, which can only be converted at maturity, while Zhu [5] presented a closed-form analytical solution for a convertible bond, which can be converted at any time on or before maturity, using the homotopy analysis method. Recently, Chan & Zhu [6] provided an approximate solution for the price of a convertible bond under the regime-switching model.

On the other hand, numerical approaches are resorted to when CBs with more complex exercise clauses need to be priced. Among them, the finite element approach [7], the finite difference approach [8] and the finite volume approach [9] have been adopted by various authors. In terms of integral equation formulations for pricing CBs, Zhu & Zhang [10] used

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https://doi.org/10.1016/j.camwa.2018.01.007
0898-1221/

Please cite this article in press as: S.-P. Zhu, et al., Pricing puttable convertible bonds with integral equation approaches, Computers and Mathematics with Applications (2018), https://doi.org/10.1016/j.camwa.2018.01.007.

a decomposition approach to obtain an integral equation formulation for pricing a vanilla convertible bond without any additional feature such as the puttability discussed in this paper.

Apart from the Black–Scholes model, there are other models having been adopted for the evaluation of CBs. For example, Brennan & Schwartz [11] proposed a stochastic interest rate model to price convertible bonds, taking the value of the issuing firm as the underlying state variable. Carayannopoulos [12] priced convertible bonds with a different stochastic interest rate model (the so-called CIR model (Cox–Ingersoll–Ross) [13]), while David & Lischka [14] adopted Vasicek's model [15]. All these models are based on an assumption that CBs are usually designed for a long time period, during which interest rate itself may be subject to changes. However, such an addition of stochastic nature of interest rate would not be necessary if one only needs to price a CB with short time to expiry. It is certainly not necessary if one aims to develop numerical approaches as their first step. Furthermore, Hung & Wang [16] used the binomial tree model to value the convertible bond, taking the risk of interest rate change as well as the default risk of the issuer into consideration, while Chambers & Lu [17] further extended Hung & Wang's work by allowing correlations among those two stochastic processes.

In addition to model complexity contributing to the pricing of CBs, various added additional rights to either or both the bond issuer and/or the bond holder, may also make the pricing problem more complicated, which demands better numerical solution approaches. For example, call and put features can be added to convertible bonds to form the so-called callable convertible bonds and puttable convertible bonds [18], respectively. A callable convertible bond is a bond in which the issuer has the right to call (repurchase) the bond from the investor for a predetermined call price within a predetermined callable period. The call feature in a convertible bond is in favor to the issuer, as if the underlying price increases significantly beyond the call price, the issuer can call back the bond. As a result, a callable convertible bond should be worth less than that of a vanilla convertible bond. A puttable convertible bond, on the other hand, allows the holder to sell the bond back to the issuer, prior to maturity, at a price that is specified at the time that the bond is issued. This price is commonly referred to as the put price [19], which is also called the strike or exercise price [3]. Obviously, the put feature benefits the holder of the bond, and hence, a puttable convertible bond trades at a higher price than that of a vanilla convertible bond.

The pricing problem of callable convertible bonds has been studied for many years. For example, Brennan & Schwartz [2] explained in theory how to price such contracts, and provided numerical solutions in their later article [20], while Bernini [21] used a binomial tree method to obtain their numerical solution. It is interesting to note that Kifer [22] presented a new derivative security called game options, similar to the callable convertible bond, which was used by Yagi & Sawaki [23] to study callable convertible bonds. There are also many references on puttable convertible bonds in the literature. For example, Nyborg [4] presented the boundary condition of puttable CBs, and checked if the boundary condition is reasonable and correct, while Lvov et al. [19] obtained the numerical solution by using Monte Carlo simulations. However, there has not been any integral equation formulation for puttable convertible bonds, which forms the base of the current research.

In this paper, we present two integral equation formulations to analyze a puttable convertible bond under the Black–Scholes model. It should be pointed out that although it is more practical to adopt a stochastic interest rate for convertible bond pricing, we assume a constant interest rate in our formulation. This is because it is more feasible to start with a simpler model when introducing a new solution approach to an already complicated problem with two free boundaries. There are two partial differential equation (PDE) systems governing the price of a puttable convertible bond, as the lifetime of a puttable CB is divided into two intervals by the time when the face value of the bond discounted by the time to expiry equals the predetermined put price. From this critical time until expiry, the price of a puttable CB is always greater than the put price so there is no financial incentive to exercise the put feature at all. Thus, the PDE system for this part should be the same as that for the vanilla CB presented in [5]. On the other hand, from the beginning of the contract until the critical time, the minimal price of puttable CB would be floored below by the put price, forming a second free boundary. Financially, the bound price is bounded below because the holder would otherwise sell the bond back to the issuer at the put price with the warranted puttability. As a result, the puttable CB can no longer be treated as a vanilla CB and another PDE system is needed with two free boundary conditions associated with the conversion and put feature, respectively.

In order to obtain the first integral equation formulation, we apply the method of incomplete Fourier transform [24] to both of the two PDE systems. However, the resulting integral equations possess a discontinuity at both of two free boundaries and they contain the first-order derivatives of the unknown free boundaries. These problems could lead to computational difficulties when the numerical results are calculated. To overcome the problems, we derive a second integral equation representation from the first integral representation.

The paper is organized as follows. In Section 2, the PDE systems governing the price of a puttable CB are established to reflect all the unique features associated with conversion and puttability at any time prior to expiry. In Section 3, the first form of integral equation is derived by using the incomplete Fourier transform, which serves as a base to obtain another integral equation representation. In Section 4, we compare our results with the known benchmarks such as the convergent results obtained with the binomial tree method. Numerical examples are presented in Section 5, followed by some concluding remarks given in the last section.

2. The model

In this section, we will establish the PDE systems to price a puttable convertible bond.

Let S be an underlying asset price and we assume that its dynamics follows the stochastic different equation:

$$dS = (r - D_0)Sdt + \sigma S dW_t, \tag{2.1}$$