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Lie-group similarity solution and analysis for fractional viscoelastic MHD fluid over a stretching sheet

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ABSTRACT

Lie-group is introduced for studying boundary layer flow and heat transfer of fractional viscoelastic MHD fluid over a stretching sheet. Fractional boundary layer equations, based on Riemann–Liouville operators, are reduced and solved numerically by Grünwald scheme approximation. Results show that the skin friction and thermal conductivity are strongly affected by magnetic field parameter, fractional derivative and wall stretching exponent. The bigger of the fractional order derivative leads to the faster velocity of viscoelastic fluids near the plate but not to hold near the outer flow. Skin friction increases with increase of magnetic field parameter M, while the heat transfer decreases. For wall stretching exponent parameter $\beta = 1.0$, the velocity profile decreases with the increase of similarity variable η. However, for $\beta = -1.5$, the velocity profile increases initially and then decreases afterwards with the biggest velocity at the interior of boundary layer.

1. Introduction

Magnetohydrodynamic (MHD) flow and heat transfer over a stretching sheet is of importance in various applications, such as annealing of copper wires, the purification of molten metals and extrusion of polymeric fluids and melts. Many researchers have made great efforts to model and analyze the transport phenomena of magnetic fluid. Pavlov was the first [1], to the authors' knowledge, to study the MHD flow due to deformation of a plane elastic surface. Chiam [2] studied the hydromagnetic flow of a Newtonian fluid over a stretching surface in the presence of a magnetic field. Chandran et al. [3] examined the effects of a magnetic field on the flow and heat transfer past a continuously moving porous plate in a stationary fluid. Vajravelu et al. [4] studied the flow and heat transfer characteristics for the electrically conducting fluid near an isothermal sheet. Mukhopadhyay et al. [5] addressed the MHD boundary layer flow past a heated stretching sheet with variable viscosity, which was assumed to linearly vary as the temperature. Wu et al. and Chen et al. [6,7] investigated the MHD flow and heat transfer with suction/injection. Andersson et al. [8] studied the MHD flow of a power-law fluid over a stretching surface in the presence of a uniform transverse magnetic field. Pop and Na [9] performed an analysis of the MHD flow over a stretching permeable surface. Liao [10] obtained the analytic solution for the MHD flow of power-law fluids by the homotopy analysis method (HAM). Chen et al. [11,12] extended the works on the MHD boundary layer flow of the power-law fluid over a moving surface with suction/injection via HAM.

It should be noted that most of the MHD fluids in industrial applications possess viscoelastic properties in nature. Many researchers paid more attention to the velocity and thermal fields of MHD viscoelastic fluid [13–18]. Siddheshwar et al. [19], Abel et al. [20], and Turkyilmazoglu [21] studied the flow and heat transfer of MHD viscoelastic fluid under various

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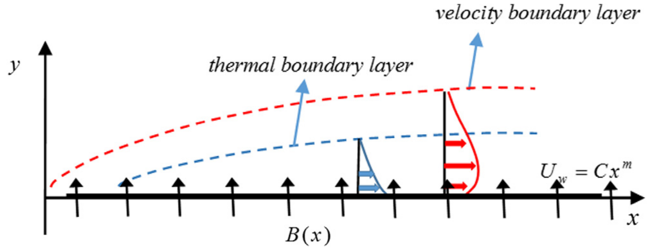

Fig. 1. Boundary layer system sketch.

physical situations. Prasad et al. [22] considered the effects of varying viscosity on MHD viscoelastic fluid over a stretching sheet. Nadeem et al. [23] carried out a numerical study on MHD boundary layer flow of a Maxwell fluid past a stretching sheet. Moreover, Li et al. [24] explored the MHD viscoelastic flow and heat transfer over a vertical stretching sheet with Cattaneo–Christov heat flux effects. In all these studies, the investigators restricted the constitutive equations to integer order derivatives.

Recently, fractional derivatives were found quite flexible in the description of the constitutive relations for the generalized non-Newtonian viscoelastic fluids. Many studies have focused on the modeling of the constitutive relationships by fractional calculus operators. And some numerical methods to solve the fractional viscoelastic fluid model and fractional partial differential equations have been proposed in literatures [25–28]. Fan et al. [29] provided a specific and efficient parameter estimation method for the fractional fractal diffusion model. Feng et al. [30] explored the Numerical methods for simulating the flow of a generalized Oldroyd-B fluid between two infinite parallel rigid plates. Hayat et al. [31] applied the fractional calculus approach to describe the modified Darcy's relationship in MHD flow of generalized Burgers' fluid. Khan et al. [32] reported an exact solution for MHD flow of a generalized Oldroyd-B fluid in a circular pipe. Liu et al. [33] dealt with the heat conduction phenomena using the fractional Cattaneoe–Christov upper-convective derivative flux model. Zhao et al. [34] formulated and derived the fractional boundary layer equations of Maxwell viscoelastic fluid over a vertical plate. Cao et al. [35] extended the works on the MHD flow and heat transfer of a fractional Maxwell viscoelastic nanofluid over a moving plate. Moreover, Lie group analysis method has been proven as an efficient approach to studying the properties of fractional differential equations and finding out the exact solutions. Additionally, in fluid mechanics, Lie group analysis was successfully applied to flow and heat transfer problems of the generalized viscoelastic fluids [36–40].

Inspired by the above investigations, this research focuses on the MHD boundary layer equations of fractional viscoelastic fluid over a stretching sheet. The fractional derivative shear stress and Fourier's law are introduced in the constitutive relations. Two nonlinear ordinary differential equations are deduced from the nonlinear governing equations by Lie group transformation. Similarity solutions are obtained by a shooting method coupled with the shifted Grünwald formula. The effects of the magnetic field parameter, the order of fractional derivative, the sheet velocity exponent, and the generalized Prandtl number on the MHD boundary layer velocity and temperature fields are analyzed.

2. Mathematical formulation

Consider boundary layer flow and heat transfer of an electrically conducting viscoelastic fluid on a stretching sheet in the power law variation $U_w = Cx^m$, where the essential features of such a flow are illustrated in Fig. 1.

The governing boundary layer equations of momentum and energy are as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{1}{\rho}\frac{\partial \tau_{xy}}{\partial y} - \frac{\sigma B(x)^2}{\rho}u \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = -\frac{1}{\rho c_p}\frac{\partial q}{\partial y}. \tag{3}$$

The boundary conditions are

$$u|_{y=0} = U_w, \quad v|_{y=0} = 0, \quad u|_{y=+\infty} = 0, \tag{4}$$

$$T|_{y=0} = T_w \quad T|_{y=+\infty} = T_\infty \tag{5}$$

where C is a positive constant, u and v are the velocity components parallel and normal, respectively, to the plate, ρ is the fluid density, σ is the electrical conductivity, $B(x)$ is an external magnetic field, and τ_{xy} is the shear stress.

As a generalization and development of integer order one, the fractional derivative indicates that the position we consider is not only depended on its nearby positions but also on the whole positions, while the integer order operator is only a local