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A delayed diffusive predator–prey system with predator cannibalism

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ABSTRACT

A diffusive predator–prey system with cannibalism and maturation delay in predator subject to Neumann boundary condition is studied in this paper. Instability and Hopf bifurcation induced by time delay are investigated. By the theory of normal form and center manifold method, the conditions for determining the bifurcation direction and the stability of the bifurcating periodic solution are derived. Some numerical simulations are given to support our results.

1. Introduction

Since the extensive existence of predation relation in nature, many researchers have studied predator–prey model by using ordinary differential equations, partial differential equations, functional differential equations and other mathematical methods. They derived some important conclusions [1–6].

A diffusive predator–prey system with Holling type-II functional response based on the classical Lotka–Volterra model is a kind of important predator–prey model, with the following form,

$$\begin{cases} \dfrac{\partial u(x,t)}{\partial t} = d_1 \Delta u + ru\left(1 - \dfrac{u}{K}\right) - \dfrac{auv}{1+ahu}, & x \in (0,\Omega),\ t > 0 \\ \dfrac{\partial v(x,t)}{\partial t} = d_2 \Delta v + v\left(\dfrac{bau}{1+ahu} - d\right), & x \in (0,\Omega),\ t > 0 \\ u_x(0,t) = v_x(0,t) = 0,\ u_x(\Omega,t) = v_x(\Omega,t) = 0, & t > 0 \\ u(x,0) = u_0(x) \geq 0,\ v(x,0) = v_0(x) \geq 0, & x \in [0,\Omega]. \end{cases} \tag{1.1}$$

Here $u(t)$ and $v(t)$ represent prey and predator densities at time t respectively. All the parameters in the model are positive. r is the growth rate of the prey. K is the carrying capacity of the prey in ecosystem. a and h represent the scalings of the predator–prey encounter rate, and h is the handling time [7,8]. b is the conversion efficiency of prey into the predator. d is the death rate of predator. d_1 and d_2 are the constant diffusion coefficient of prey and predator. The boundary condition is Neumann boundary condition. Based on the assumption the region $[0, \Omega]$ is closed, with no prey and predator species entering and leaving the region at the boundary. $\Omega = l\pi$, where $l > 0$.

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Many scholars have studied system (1.1). Yi et al. studied the Hopf and steady state bifurcation of system (1.1), and they showed that spatially nonhomogeneous periodic orbits and nonhomogeneous steady state solutions exist [1]. In [9], Peng and Shi proved the non-existence of non-constant positive steady state solutions when the interaction between the predator and the prey is strong for system (1.1). Ko and Ryu investigated the existence and non-existence of non-constant positive steady-state solutions for system (1.1) incorporating a constant proportional prey refuge [10]. Chen et al. analyzed stability and Hopf bifurcation for system (1.1) by introducing a time delay [11].

Cannibalism is a widespread phenomenon in nature [12,13]. Predator may kill each other for food, increasing the death rate of predator. In [14], Sun et al. introduced a predator's cannibalism term into system (1.1), with the following form,

$$\begin{cases} \dfrac{\partial u(x,t)}{\partial t} = d_1 \Delta u + ru\left(1-\dfrac{u}{K}\right) - \dfrac{auv}{1+ahu}, \ x\in(0,\Omega),\ t>0 \\ \dfrac{\partial v(x,t)}{\partial t} = d_2 \Delta v + v(\dfrac{bau}{1+ahu} - d) - \dfrac{av^2}{1+ahu}, \ x\in(0,\Omega),\ t>0 \\ u_x(0,t)=v_x(0,t)=0, u_x(\Omega,t)=v_x(\Omega,t)=0, \ \ t>0 \\ u(x,0)=u_0(x)\geq 0, v(x,0)=v_0(x)\geq 0, \ \ x\in[0,\Omega]. \end{cases} \tag{1.2}$$

The term $av^2/(1+ahu)$ represents predator cannibalism. In [14], authors studied spatial pattern of system (1.2). They show that predator cannibalism can affect the spatial pattern formation.

In recent years, predator–prey systems with time delay derive great attention, since delayed predator–prey systems can exhibit rich dynamics [15–19]. Time delay may affect the stable or unstable outcome of prey densities due to predation. In delayed predator–prey models, time delay occurs in maturation time, capturing time, gestation time or others. To consider the effect of time delay on the system (1.2), we study the following system

$$\begin{cases} \dfrac{\partial u(x,t)}{\partial t} = d_1 \Delta u + ru\left(1-\dfrac{u}{K}\right) - \dfrac{auv}{1+ahu}, \ x\in(0,\Omega),\ t>0 \\ \dfrac{\partial v(x,t)}{\partial t} = d_2 \Delta v + v\left(\dfrac{bau(t-\tau)}{1+ahu(t-\tau)} - d - \dfrac{av}{1+ahu}\right), \ x\in(0,\Omega),\ t>0 \\ u_x(0,t)=v_x(0,t)=0, u_x(\Omega,t)=v_x(\Omega,t)=0, \ \ t>0 \\ u(x,\theta)=u_0(x,\theta)\geq 0, v(x,\theta)=v_0(x,\theta)\geq 0, \ \ x\in[0,\Omega], \theta\in[-\tau,0] \end{cases} \tag{1.3}$$

where $\tau \geq 0$ is time delay occurs in the gestation time of predator. The aim of this paper is to consider the stability and Hopf bifurcation of system (1.3).

The rest of this paper is organized as follows. In Section 2, we study the existence of coexisting equilibrium. In Section 3, we study the delayed model, including local stability, Hopf bifurcation at positive equilibrium. In Section 4, we study the stability and direction of Hopf bifurcation. In Section 5, we give some numerical simulations to illustrate the theoretical results. In Section 6, we give a brief conclusion.

2. Equilibrium analysis

The equilibria of system (1.3) is the roots of the following equations:

$$\begin{aligned} & ru\left(1-\frac{u}{K}\right) - \frac{auv}{1+ahu} = 0 \\ & v\left(\frac{bau}{1+ahu} - d - \frac{av}{1+ahu}\right) = 0. \end{aligned} \tag{2.1}$$

Obviously, system (1.3) has boundary equilibria $(0,0)$ corresponding to total extinction of the species, and $(K,0)$ corresponding to extinction of the predator. In this paper, we mainly focus on the coexisting equilibrium of the system (1.3). Now we discuss the existence of coexisting equilibrium point. Suppose (u_*, v_*) is coexisting equilibrium point of the system (1.3). Solving the first equation in Eq. (2.2) yields $v_* = r/a(1+ahu_*)(1-u_*/K)$. Obviously, $v_*>0$ implies $0<u_*<K$. Subtracting second equation from the first equation in (2.2), yields

$$h(u) = ahru^2 - (aK(h(d+r)-b)-r)u - K(d+r) = 0.$$

Then system (2.2) has coexisting equilibrium (u_*, v_*) if and only if $h(u)=0$ has root u_* such that $0<u_*<K$. Clearly, $h(0) = -K(d+r) < 0$ and $h(K) = K(a(b-dh)K-d)$. Hence, if

$(\mathbf{H_0})$ $\quad d < a(b-dh)K$