ARTICLE IN PRESS

Engineering Science and Technology, an International Journal xxx (2018) xxx–xxx

HOSTED BY

ELSEVIER

Contents lists available at ScienceDirect

Engineering Science and Technology, an International Journal

journal homepage: www.elsevier.com/locate/jestch

Full Length Article

Advanced sliding mode control techniques for Inverted Pendulum: Modelling and simulation

Saqib Irfan [a], Adeel Mehmood [a], Muhammad Tayyab Razzaq [b], Jamshed Iqbal [c,d,*]

[a] Department of Electrical Engineering, COMSATS University Islamabad, Pakistan
[b] Department of Physics, Quaid-i-Azam University, Islamabad, Pakistan
[c] Electrical and Computer Engineering Department, University of Jeddah, Saudi Arabia
[d] Department of Electrical Engineering, FAST National University of Computer and Emerging Sciences, Islamabad, Pakistan

ARTICLE INFO

Article history:
Received 20 December 2017
Revised 25 May 2018
Accepted 12 June 2018
Available online xxxx

Keywords:
Inverted Pendulum (IP)
Sliding mode control
Lyapunov stability

ABSTRACT

Numerous practical applications like robot balancing, segway and hover board riding and operation of a rocket propeller are inherently based on Inverted Pendulum (IP). The control of an IP is a sophisticated problem due to various real world phenomena that make it unstable, non-linear and under-actuated system. This paper presents a comparative analysis of linear and non-linear feedback control techniques based on investigation of time, control energy and tracking error to obtain best control performance for the IP system. The implemented control techniques are Linear Quadratic controller (LQR), Sliding Mode Control (SMC) through feedback linearization, Integral Sliding Mode Control (ISMC) and Terminal Sliding Mode Control (TSMC). Considering cart position and pendulum angle, the designed control laws have been subjected to various test signals so as to characterize their tracking performance. Comparative results indicate that ISMC gives a rise time of 0.6 s with 0% overshoot and over-performs compared to other control techniques in terms of reduced chattering, less settling time and small steady state error.

1. Introduction

Recent advancements in the domain of control systems are completely reshaping mechatronics [1] and robotic systems [2]. Inverted Pendulum (IP) is a challenging problem studied in the field of control systems theory. It is a highly non-linear, unstable and under-actuated Multiple Input and Multiple Outputs (MIMO) mechanical system [3]. Consequently, such a system, requiring a sophisticated control law [4], is considered as a benchmark to develop the ideas relevant to the robust systems to characterize and compare the performance of the classical and modern control strategies on a large scale. It has wide range of industrial applications e.g. two wheeled self-balancing vehicles (seg-way), rockets, guided missiles, intelligent robots, and other systems exhibiting crane model. Various control strategies have been implemented on IP. Trivial control algorithms like Proportional Integral Derivatives (PID) [5] are not well suited and incapable of handling for such a complex and non-linear system because they cannot handle inherent uncertainties and disturbances [6]. The robustness of the system decreases with parametric and structural uncertainties consequently making tuning of gains in PID control law a very itchy task [7]. Thus, robust control laws [8] are needed to achieve a high level of precision and accuracy resulting in a more reliable and flexible system to converge the state trajectories into the system in finite time. These challenges highlight the role of more advanced and sophisticated control strategies like Linear Quadratic Regulator (LQR) and Sliding Mode Control (SMC) or its variants, which can provide a systematic way to accurately track the desired trajectories [9].

LQR is a purely linear control technique used for the linear systems while SMC is a robust control technique which deals with the complex systems where uncertainties and disturbances are present. SMC, Integral SMC (ISMC) [10] and Terminal SMC (TSMC) are robust control techniques so, there is always an inconsistency between mathematical and actual model for designing a controller [11]. Unknown external disturbances like matched and unmatched uncertainties are the source of discrepancies between the actual and mathematical model of the system [12,13].

When a pendulum moves in the upright position, stability requirements necessitate the use of robust control technique which should be capable of dealing with the fast dynamics of the

* Corresponding author at: Electrical and Computer Engineering Department, University of Jeddah, Saudi Arabia.

E-mail address: jmiqbal@uj.edu.sa (J. Iqbal).

Peer review under responsibility of Karabuk University.

https://doi.org/10.1016/j.jestch.2018.06.010
2215-0986/© 2018 Karabuk University. Publishing services by Elsevier B.V.

Please cite this article in press as: S. Irfan et al., Advanced sliding mode control techniques for Inverted Pendulum: Modelling and simulation, Eng. Sci. Tech., Int. J. (2018), https://doi.org/10.1016/j.jestch.2018.06.010

system. The chattering phenomena present in first order SMC, law can be handled by higher order sliding mode control technique which also improves the system performance [14].

In this paper a comparative analysis of control strategies i.e. LQR, SMC, ISMC, TSMC have been presented. All these algorithms have been implemented on IP. The comparison is carried out based on various parameters like time, control energy and chattering phenomena. The results depict efficacy of ISMC over LQR, SMC and TSMC. The remaining paper is organized as follows: Section 2 derives linear and non-linear model of IP. Section 3 details the control techniques under study while, Section 4 presents the results and discussion of the implemented control strategies. Finally, Section 5 presents the conclusion of this paper.

2. Methodology – Mathematical modelling

Mathematical model is required to design the control law. Therefore, Newton law based model of the IP has been derived. The IP consists of a moveable cart rail system and a swing-able pole connected to the cart as shown in Fig. 1. Cart position is controlled with DC motor.

The non-linear mathematical model of the IP is derived using the Newton law approach. Vertical force does not affect the cart position and the horizontal movement is controlled by the forces applied through DC motors [15,16]. The obtained non-linear mathematical model of system is given by (1) and (2). The nomenclature is explained in Table 1, and system specifications are provided in Table 2.

$$(M+m)\ddot{x} + mL\ddot{\theta}\cos\theta - -ml\dot{\theta}^2\sin\theta + B\dot{x} = F \tag{1}$$

$$(I+ml^2)\ddot{\theta} + mgl sin\theta = -ml\ddot{x}cos\theta \tag{2}$$

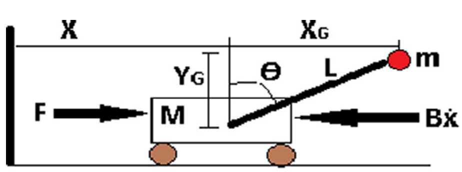

Fig. 1. Physical model of IP.

Table 1
System states and parameters.

Symbol	Description
$B\dot{x}$	Cart friction
F	External force for horizontal movement
$x, \dot{x}, \ddot{x}$	Cart position, velocity, acceleration respectively
M	External pole moment
m	Pole friction moment
$\theta, \ddot{\theta}$	Angular velocity and angular acceleration
l	Length of the pendulum

Table 2
Assigned values to system.

Symbol	Quantity/Meaning	Value	Unit
m	Pendulum mass	2.4	kg
M	Cart mass	0.23	kg
I	Inertia	0.38	kg/m^2
g	Gravity	9.8	m/s^2
$B\dot{x}$	Friction of cart	0.005	N
l	Length of pendulum	45	cm
L	Cart length	91	cm

To implement LQR [17] on this set of equations, we need to linearize the non-linear terms $\dot{\theta}^2$ When pendulum is stable at $\theta \cong 0, \dot{\theta}^2 \cong 0$ and $\cos(0) = 1$, the mathematical model is reduced to (3) and (4).

$$(M+m)\ddot{x} + B\dot{x} - ml\ddot{\theta} = F \tag{3}$$

$$(I+ml^2)\ddot{\theta} - mgl\theta = ml\ddot{x} \tag{4}$$

The transfer functions of the cart position and angle of pendulum is given as,

$$\frac{X(s)}{U(s)} = \frac{\frac{(I+ml^2)s^2 - mgl}{q}}{s^4 + \frac{b(I+ml^2)}{q}s^3 - \frac{(M+m)mgl}{q}s^2 - \frac{bmgl}{q}s} \tag{5}$$

$$\frac{\theta(s)}{U(s)} = \frac{\frac{ml}{q}s}{s^3 + \frac{b(I+ml^2)}{q}s^2 - \frac{(M+m)mgl}{q}s - \frac{bmgl}{q}} \tag{6}$$

where, q is defined as follow:

$$q = [(M+m)(I+Ml^2) - (ml^2)] \tag{7}$$

Converting (5) and (6) to the equivalent state space form given as,

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{\theta} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -0.0194 & 0.2188 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -0.0128 & 6.5848 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \theta \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0.3887 \\ 0 \\ 0.2552 \end{bmatrix} u(t) \tag{8}$$

The output matrix can be written as,

$$y(t) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \theta \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} u(t) \tag{9}$$

The system has four poles with two in the right half plane which makes the system unstable. Therefore, a linear controller needs to be designed to force the poles in the left half plane. The calculated open loop poles are located at: S = 0–5.6041 5.5651–0.1428.

3. Methodology – Control techniques

In this section, LQR based control technique, feedback linearization based on SMC, ISMC and TSMC are described in details. On the basis of system performance, parameters like setting time, rise time, steady state error and overshoot are calculated.

3.1. Linear Quadratic Regulator (LQR)

LQR is a linearized and optimal control technique which provides optimum gains for the systems. It is more suitable for the linear systems having no uncertainties or disturbances. The major benefit of this technique is that it gives the gains to minimize the cost function [18,19] represented by (10). For an nth order system the general cost function of LQR is given as,

$$J = \int_0^\infty [x^T(t)Q(t)x(t) + U^T(t)R(t)U(t)]dt \tag{10}$$

where, $Q \in \mathcal{R}^{n\times n}$ is positive definite or positive semi definite Hermitian matrix (or real symmetric matrix), $R \in \mathcal{R}^{r\times r}$ is a positive definite Hermitian matrix (or real constant number), $S \in \mathcal{R}^{n\times n}$ is a positive definite Hermitian matrix (or real symmetric matrix).

The LQR gain is computed as,

$$K = R^{-1}B^TP \tag{11}$$

Please cite this article in press as: S. Irfan et al., Advanced sliding mode control techniques for Inverted Pendulum: Modelling and simulation, Eng. Sci. Tech., Int. J. (2018), https://doi.org/10.1016/j.jestch.2018.06.010