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A Lagrangian heuristic for an integrated lot-sizing and fixed scheduling problem

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ABSTRACT

This paper presents a novel approach for solving an integrated production planning and scheduling problem. In theory as well as in practice, because of their complexity, these two decision levels are most of the time treated sequentially. Scheduling largely depends on the production quantities (lot sizes) computed at the production planning level and ignoring scheduling constraints in planning leads to inconsistent decisions. Integrating production planning and scheduling is therefore important for efficiently managing operations. An integrated model and an iterative solution procedure were proposed in earlier research papers: The approach has limitations, in particular when solving the planning problem. In this paper, a new formulation is proposed to determine a feasible optimal production plan, i.e. lot sizes, for a fixed sequence of operations on the machines when setup costs and times are taken into account. Capacity constraints correspond to paths of the conjunctive graph associated to the sequence. An original Lagrangian relaxation approach is proposed to solve this NP-hard problem. A lower bound is derived and an upper bound is calculated using a novel constructive heuristic. The quality of the approach is tested on numerous problem instances.

1. Introduction

Production decisions in a manufacturing environment aim at determining the most effective way to use resources for the production of items while satisfying customers' requirements. These decisions are generally separated into two levels: A planning level and a scheduling level. At the planning level, the objective is to determine a production plan, i.e. production quantities for every period of the horizon, that satisfy the demands and minimize the different costs (production costs, setup costs, holding costs . . .). These production quantities correspond to the sizes of the lots processed in the shop floor. At the scheduling level, these lots are sequenced on the resources.

However, in practice and in theory, lot-sizing and scheduling decisions are still often taken sequentially. Mathematical planning models take into account aggregate capacity constraints. They do not guarantee that the proposed production plan is feasible when it is forwarded to the scheduling level, i.e. there exists a schedule which allows lots of the production plan to be produced on time (see Dauzère-Pérès & Lasserre, 2002). If capacity is overestimated, capacity constraints in the production plan do not represent the reality in terms of resource availability in the workshop. Thus, the estimated time for producing an item will be lower than the actual production time, which causes delays and dissatisfied customers. On the opposite, when capacity is underestimated, items will be completed earlier than planned, leading to large work-in-process inventories. It seems thus important to simultaneously deal with planning and scheduling decisions in order to determine a feasible and relevant production plan. This is particularly true when items are produced in lots.

Many studies have addressed production planning or scheduling problems but very little attention has been given to the integration of these two decision levels. It should be noted that one of the primary difficulties in solving integrated problems is the nature of the involved decision variables. The planning level determines flows of products and typical models involve continuous variables, whereas models at the scheduling level include discrete variables for defining sequences of products on resources.

The most common approach used for production planning in practice is MRP (*Material Requirements Planning*). Its assumptions are that production capacity is unlimited and lead times are generally assumed to be fixed. In MRP II (*Manufacturing Resource Planning*), aggregate capacities are taken into account but are usually not enough to ensure consistency between planning and scheduling decisions.

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Lot-sizing problems have been studied in the literature extensively. However, most lot-sizing problems consider aggregate capacity constraints, which make sense when each lot is processed independently on the resources and lots do not have to be sequenced, i.e. if each lot could start simultaneously on all its resources. In this case, the total workload on the machines is then relevant to consider for capacity restrictions. However, when production is by lots, the duration of an elementary operation of a lot is usually no longer negligible. The sequencing of lots on the machines should not be ignored. Dauzère-Pérès and Lasserre (1994, 2002) and Ouenniche, Boctor, and Martel (1999) study the impact of sequencing decisions on multi-item lot-sizing and scheduling problems. Small time-bucket lot-sizing problems (Drexl & Kimms, 1997) consider short time periods and the sequencing of lots. A basic small bucket problem is the Discrete Lot-Sizing and scheduling Problem (DLSP) (Fleischmann, 1990). The main drawback of the DLSP is the so-called *all-or-nothing* production, which means that only one item may be produced per period and, in that case, the production quantity is equivalent to using full capacity. This drawback is overcome in the Continuous Setup Lot-Sizing Problem (CSLP), but still only one item per period may be produced. In the Proportional Lot-Sizing and scheduling Problem (PLSP) (Drexl & Haase, 1995), the remaining capacity in a given period is used for scheduling a second item. These models allow simultaneous lot sizing and scheduling but limit the number of products to be manufactured in one period. The General Lot-Sizing and scheduling Problem (GLSP) (Fleischmann & Meyr, 1997) takes into account multiple products but features a single capacitated machine. An extension of these problems to multiple machine environments is proposed by Kimms (1999) for the PLSP and Fandel and Stammen-Hegene (2006) for the GLSP. Another class of problems considers lot-sizing and scheduling problem with sequence-dependent setup costs and/or setup times (see for example Almada-Lobo, Klabjan, Carravilla, & Oliveira, 2007, 2010; Almeder, Klabjan, Traxler, & Almada-Lobo, 2015; Gupta & Magnusson, 2005; Haase & Kimms, 2000; Menezes, Clark, & Almada-Lobo, 2010; Meyr, 2000). The main characteristic of these models is that they consider a single resource. Multiple resources in parallel are considered in James and Almada-Lobo (2011) with sequence-dependent setup times and costs, but their model is still far from taking complex job-shop scheduling constraints into account.

Some researchers proposed different approaches for solving the integrated planning and scheduling problem. Dauzère-Pérès and Lasserre (1994, 2002) were among the first to propose an integrated planning and scheduling model. Their models consider complex scheduling problems and the exact capacity of the shop floor is taken into account. They propose a solution procedure that iterates between a planning module and a scheduling module. In the scheduling module, an optimal schedule is determined for fixed lot sizes while, in the planning module, the best lot sizes are calculated for a fixed sequence of operations. This procedure allows a feasible production plan to be determined. Sikora, Chhajed, and Shaw (1996) describe an integrated approach where lot-sizing and sequencing decisions are taken separately. Their approach solves the problem period by period, from the first to the last period of the horizon. At each period, the procedure iterates between sequencing the lots on the machines and determining the lot sizes for a single bottleneck machine (using a modified Silver-Meal heuristic) until the capacity of the period is used as much as possible. Their approach is applied on data of a specific manufacturing line which produces Printed Circuit Boards. Our approach aims at solving the problem globally, and interactions between different periods are allowed. Giglio and Minciardi (2002) propose a formulation for integrating mid-term production planning with short-term production scheduling. Their model also has two types of decision variables: Continuous variables for the lot-sizing and the timing of operations, and combinatorial variables for the assignment and the sequencing of operations over resources. Their approach assumes that all combinatorial variables are fixed, and the model is solved using standard tools. Discrete variables associated to setup costs and times are considered in our approach, and we propose a Lagrangian heuristic to solve large instances of the problem. Jodlbauer (2006) develops an approach to determine the schedule, lot sizes and sequences for a multi-item capacitated production system, based on a specific property of the setup cost function, which enables the number of setup activities to be replaced in the integer model. Backlogging is not allowed in his model and only one machine is considered.

Metaheuristic may also be used to solve the integrated problem. Kim, Park, and Ko (2003) propose an approach that simultaneously solves process planning and scheduling problems in flexible job shops using a symbiotic evolutionary algorithm. This problem is not really a production planning problem since production quantities are not determined and the objective is to minimize the absolute deviation of machine loads, makespan and mean flow time. Zhang and Yan (2005) propose a hybrid genetic algorithm for an integrated job-shop production planning and scheduling problem where the sequence of operations is given. However, their model is different than ours on various aspects, in particular periods are again independent. Moreover, their algorithm is not compared to an exact method, is only tested on very few problems instances and is slow compared to the one proposed in this paper.

We propose a new mathematical model where the objective is to minimize the sum of the production, inventory and setup costs subject to detailed scheduling constraints for a given sequence of operations on the resources. This NP-hard problem (see Section 2.1) is motivated by at least two reasons. First, this is actually a practical problem in some process industries where a preferred sequence of products can be used. The second and main motivation is to use the approach presented in this paper as building block in an iterative procedure that solves the planning problem for a fixed sequence and the scheduling problem for fixed lot sizes. We actually have successfully done it in Gomez-Urrutia, Aggoune, and Dauzère-Pérès (2014), in which the approach proposed in the present paper is used, and not presented, to solve the integrated problem.

The capacity constraints in our new model correspond to paths of the conjunctive graph associated to the sequence. A Lagrangian heuristic is used to solve this problem where capacity constraints are relaxed. The originality of our approach is twofold. First, since there exists an exponential number of capacity constraints, only Lagrangian multipliers related to the most violated constraints are explicitly updated. Second, the procedure used to construct a feasible solution is significantly different from the classical smoothing heuristics used in previous Lagrangian heuristics for lot-sizing problems.

The remainder of the paper is organized as follows. Section 2 recalls the mathematical model of Dauzère-Pérès and Lasserre (*DPL*) (Dauzère-Pérès & Lasserre, 1994, 2002) with setup costs and times, and presents a new model for a given sequence of jobs on the resources. The Lagrangian relaxation heuristic used for solving the integrated problem is detailed in Section 3. Experimental results are presented and analyzed in Section 4. The paper ends with some conclusions.

2. Problem formulations

2.1. Problem description

As in Dauzère-Pérès and Lasserre (2002), the production of N different items has to be planned on a horizon of T periods to satisfy demands at the lowest cost, where D_{il} denotes the demand of item i at the end of period l. Detailed scheduling constraints are explicitly considered, namely each item has to be processed in a series of operations (routing) before being completed. Although this assumption could be relaxed by creating additional variables, we suppose that at most one lot of each item is associated to each period. Operations