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Decision Support

Modeling assignment-based pairwise comparisons within integrated framework for value-driven multiple criteria sorting

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ABSTRACT

We introduce a new preference disaggregation modeling formulations for multiple criteria sorting with a set of additive value functions. The preference information supplied by the Decision Maker (DM) is composed of: (1) possibly imprecise assignment examples, (2) desired class cardinalities, and (3) assignment-based pairwise comparisons. The latter have the form of imprecise statements referring to the desired assignments for pairs of alternatives, but without specifying any concrete class. Additionally, we account for preferences concerning the shape of the marginal value functions and desired comprehensive values of alternatives assigned to a given class or class range. The exploitation of all value functions compatible with these preferences results in three types of results: (1) necessary and possible assignments, (2) extreme class cardinalities, and (3) necessary and possible assignment-based preference relations. These outputs correspond to different types of admitted preference information. By exhibiting different outcomes, we encourage the DM in various ways to enrich her/his preference information interactively. The applicability of the framework is demonstrated on data involving the classification of cities into liveability classes.

1. Introduction

Multiple criteria sorting (ordinal classification) involves the assignment of a set of alternatives evaluated using a set of criteria to one or several homogeneous classes. Despite being closely related to clustering (Meyer & Olteanu, 2013) and finding ordered segments (Chen, Cheng, & Hsu, 2013), this type of problem differs from both of them. On the one hand, clusters are not ordered, whereas classes are given in a preference order. On the other hand, segments do not need to be defined a priori, which is the case for classes. In any case, such discrimination among two or more ordered and pre-defined sets of alternatives is at the core of various real-world decision problems. Some recent sorting applications concern energy and electricity market (Diakoulaki, Zopounidis, Mavrotas, & Doumpos, 1999; Mavrotas, Diakoulaki, & Capros, 2003), climate change (Diakoulaki & Hontou, 2003), economy and finance (Doumpos & Zopounidis, 2011), stock portfolio selection (Xidonas, Mavrotas, & Psarras, 2009), cancer care (Belacel & Boulassel, 2000), airline market (Norese & Carbone, 2014), land-management (Macary, Almeida Dias, Figueira, & Roy, 2014), urban and territorial projects (Abastante, Bottero, Greco, & Lami, 2014), accreditation systems (Siskos, Grigoroudis, Krassadaki, & Matsatsinis, 2007), and tourism (Mailly, Abi-Zeid, & Pepin, 2014).

In this paper, the multiple criteria sorting model used to work out a recommendation is a set of value functions. Multi-Attribute Value Theory is a well established theory considering compensatory preference models that represent how DMs account for trade-offs among criteria. Such models are widely used and appreciated by the Multiple Criteria Decision Aiding community for their relatively small computational effort and easy interpretation. Using additive value functions requires specification of the parameters related to the formulation of marginal value functions. These parameters follow either directly or indirectly from preference information provided by the DM. The former involves direct specification of some parameter values. The latter concerns some examples of holistic or criterion-specific judgments, or requirements with respect to the delivered recommendation. This information is subsequently employed to induce values of the compatible preference model parameters which are able to restore the DM's exemplary judgments or requirements. Such indirect elicitation is usually called disaggregation.

In the last decades, methods that require indirect, imprecise, and incomplete preference statements of the DM are prevailing. In fact, several value-based disaggregation sorting methods have been already proposed in the literature. They require the DM to express her/his preferences by providing a set of assignment examples on a subset of alternatives (s)he knows

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relatively well, called reference alternatives. When using such indirect preference information, there exist multiple (usually, infinitely many) compatible instances of the preference model. Various methods handle this ambiguity in different ways. Some of them, e.g., Bous, Fortemps, Glineur, and Pirlot (2010), Devaud, Groussaud, and Jacquet-Lagreze (1980), Doumpos and Zopounidis (2007), and Greco, Kadziński, and Słowiński (2011), select a single compatible value function, thus, providing precise assignment of alternatives. Other approaches, e.g., Greco, Mousseau, and Słowiński (2010), Kadziński and Tervonen (2013), and Köksalan and Bilgin Özpeynirci (2009), take into account all compatible value functions, and investigate the spaces of consensus and disagreement between recommendation suggested by these functions. These approaches are known under the name of Robust Ordinal Regression. The results of Robust Ordinal Regression are materialized with the possible and necessary assignments, that is sorting recommendations confirmed by all or at least one compatible value function, respectively. A recent study in Doumpos, Zopounidis, and Galariotis (2014) compares experimental results on the relationship between the outcomes of a single decision model (additive value function) and the ones from the whole set of compatible model instances.

The type of admitted preference information and elements of responses obtained by the DMs, have a great impact on the consistency between value system of the stakeholders, the evolution of the decision process and recommendation of a specific decision. Nowadays, the types of admitted preference information, models, procedures, and provided results are more often perceived as a communication and reflection tool. In this spirit, the recent trend in Multiple Criteria Decision Aiding consists in accounting for types of preference information which have not received due attention in existing methods, as well as conducting diversified robustness analysis for the delivered results. Using new types of preference information increases the flexibility of the interactive procedure, thus enabling the consideration of any preference information coming from the DM. The latter aims at increasing the range of tools that can be used for looking more thoroughly into the problem, by exploring, interpreting, or testing scenarios. When it comes to recently proposed new types of preference information, let us recall desired class cardinalities, e.g., “we wish to accept at most 10 candidates” or “we need to reject at least 30 applications” (Kadziński & Słowiński, 2013; Mousseau, Dias, & Figueira, 2003), which now can be employed along with the traditionally used assignment examples.

As far as robustness analysis of sorting recommendation is concerned, apart from the already mentioned necessary and possible assignments, one has recently proposed to consider three types of results:

- assignment-based preference relations, which admit the comparison of a sorting recommendation for pairs of alternatives (Kadziński & Tervonen, 2013),
- class acceptability indices representing the shares of compatible preference model instances assigning an alternative to a particular class (Kadziński & Tervonen, 2013), and
- recommendation obtained with a value function which is representative for the whole set of compatible value functions (Kadziński, Greco, & Słowiński, 2013).

This paper can be seen as an inherent part of the above mentioned trend in Multiple Criteria Decision Aiding with the following threefold aim.

First of all, we introduce the new type of indirect preference information for sorting problems in the form of assignment-based pairwise comparisons of alternatives. Indeed, people are used to refer to such comparisons in their judgments. In many real-world decision situations, they use statements such as “*a* should be assigned to a class at least as good as *b*”, “there is a difference of at least two classes between *c* and *d*”, “*e* is better than *f* by at most one class only”, or “*g* and *h* represent the same class”. These are imprecise preference statements, which refer to the desired assignments for pairs of alternatives, but without specifying any concrete class. Note that when using such expressions, people do not rate a given alternative individually as in the assignment examples, but rather confront alternatives “one vs. one”. Nevertheless, the purpose of these statements is not to rank the alternatives, but rather to enable their comparison in terms of the sorting problem.

Furthermore, as mentioned before, the authors of Kadziński and Tervonen (2013) provided procedures for comparing sorting recommendation for pairs of alternatives. Precisely, they introduced the necessary and possible assignment-based preference relations corresponding to such results as, “irrespective of the compatible model instance, the class of alternative *a* is never worse than the class of *b*” or “there is at least one compatible model instance that assigns *a* to a class at least as good as *b*”. Such a recommendation involving all compatible preference model instances and referring directly to pairs of alternatives is not possible when using the necessary and possible assignments only. Given a framework for comparing pairs of alternatives at the output, it is even more justified to allow providing pairwise comparisons at the input of the method too.

Moreover, specification of the assignment-based pairwise comparisons of the alternatives allows to address one of the commonly acknowledged disadvantages of using some traditional disaggregation methods. Very often, the ranges of possible assignments for the alternatives are rather wide and there exist significant subsets of alternatives possibly assigned to the same class range (see, e.g., Greco, Kadziński, Mousseau, & Słowiński, 2012; Kadziński et al., 2013). Accounting for the assignment-based pairwise comparisons reduces the set of compatible preference model instances, thus making the possible assignments more precise and diversifying the recommendation obtained for different alternatives.

The second aim of the paper is to provide a framework for incorporating a number of preference modeling approaches into a single modeling approach capturing preference information given in different forms. These include assignment examples, assignment-based pairwise comparisons, and desired class cardinalities. However, we additionally account for other types of preference information concerning the shape of the marginal value functions (e.g., concavity or convexity, interval estimates of relative values, and intensities of preference) and newly introduced desired comprehensive values of alternatives assigned to a given class or class range (e.g., “alternatives assigned to class at most medium should have value not greater than 0.4” or “the difference of values between alternatives assigned to class good and bad should be at least 0.7”). We believe that such desired values are easier to provide for the DMs than, e.g., the range of variation of piecewise linear marginal value functions, and may be appreciated by some DMs also from the point of view of interpretability of the results. Accounting for all these preference statements, we provide a flexible modeling framework that incorporates a wide spectrum of indirect and imprecise preference information coming from the DM. Obviously, the ultimate goal of using all this preference information consists in applying the inferred compatible preference model on the whole set of alternatives.

In that respect, the third aim of the paper is to provide a framework for deriving a variety of results stemming from robustness analysis, including necessary and possible assignments, necessary and possible assignment-based preference relations, and extreme class cardinalities. These outputs correspond to different types of admitted preference information, i.e., assignment examples, assignment-based pairwise comparisons, and desired class cardinalities. In this way, the preference information of each type is reproduced in the respective outcome. The DM may also observe the impact of her/his preferences on the sorting recommendation concerning the whole set of alternatives (in case of assignments), all pairs of alternatives (in case of assignment-based preference relations), and all classes (in case of