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Reliability analysis of a single warm-standby system subject to repairable and nonrepairable failures

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ABSTRACT

An n-unit system provisioned with a single warm standby is investigated. The individual units are subject to repairable failures, while the entire system is subject to a nonrepairable failure at some finite but random time in the future. System performance measures for systems observed over a time interval of random duration are introduced. Two models to compute these system performance measures, one employing a policy of block replacement, and the other without a block replacement policy, are developed. Distributional assumptions involving distributions of phase type introduce matrix Laplace transformations into the calculations of the performance measures. It is shown that these measures are easily carried out on a laptop computer using Microsoft Excel. A simple economic model is used to illustrate how the performance measures may be used to determine optimal economic design specifications for the warm standby.

1. Introduction

The analysis of systems configured with warm standby components as a way to improve system reliability and availability has been of considerable research interest. The early work on the analysis of systems with warm standbys was strongly influenced by three papers. Gnedenko (1965) investigated an n-component system with one warm standby and repair facility. Under the assumptions that the operating components and warm standby had exponential lifetimes and the distribution of time to perform repair was arbitrary, Gnedenko determined the Laplace transformation of the distribution of time until first system failure, the mean time until first system failure, and explored a number of limit theorems. Gopalan (1975) extended this result by assuming a single operating unit and $n-1$ warm standbys with one repair facility, and constructed the Laplace transformation of system availability and system reliability. Gopalan applied these results to the special cases of one and two warm standbys. Subramanian, Venkatakrishnan, and Kistner (1976) changed Gopalan's model by allowing r repair facilities and assumed that the operating system had an arbitrary life distribution, while the warm standbys had an exponential distribution of time until failure, and the repair time distribution was also exponential. By viewing the stochastic process defined by the number of failed standbys at time t as a birth-and-death process, they derived a system of equations to determine the mean time until system failure and system availability.

The study of systems configured with warm standbys has accelerated in recent years. Wang, Lai, and Ke (2004) used the simplification of exponential unit lifetimes and repairs to study K out of $M+W$ systems, where M is the number of operating units and W is the number of warm standbys. Srinivasan and Subramanian (2006) considered the case of a three unit system with two warm standbys having lifetimes with exponential distributions. Under the assumption that the operating unit's lifetime and the repair time had arbitrary distributions, they constructed expressions for the system reliability and point availability; however, computational efficiency was a question left for future research. Papageorgiou and Kokolakis (2010) considered two-unit parallel systems with multiple warm standbys without the possibility of unit repair. They determine an expression for system reliability recursively based on the number of warm standbys. Amari, Pham, and Misra (2012) examined a k-out-of-n warm standby system without repair. Under the assumption of exponential unit lifetimes, they constructed an expression for the distribution of system lifetime as well as several other reliability measures. Eryilmaz (2013) analyzed a k-out-of-n system without repair equipped with a single warm standby, and derived an explicit expression for the system reliability. Additional active research streams which study systems provisioned with warm standby units include the analysis of general coherent systems (Eryilmaz, 2011), k-out-of-n systems (Amari & Pham, 2010, Eryilmaz, 2013), and other specified system configurations (Bulama, Yusuf, & Bala, 2013), the analysis of phased missions (Levitin, Xing, & Dai, 2013, Mohammad et al., 2013), and the impacts of dedicated repairmen (Vanderperre & Makhanov, 2013), multiple warm standbys (Wang, Yen, & Fang, 2012), and imperfect switching (Yuan & Meng, 2011, Yun & Cha, 2010).

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Two other works warrant special consideration because of the use of distributions of phase type. Perez-Ocon and Montoro-Cazorla (2006) assumed a one-unit system with $(n-1)$ warm standbys, where the operating unit's lifetime and the repair time have distributions of phase type and the warm standbys have exponential lifetimes. By modeling the system as a level-dependent quasi-birth-and-death process, they were able to show that the system operational periods and repair periods follow distributions of phase type, and developed computationally tractable expressions for system reliability, availability, and other measures of system performance. Ruiz-Castro and Fernandez-Villodre (2012) examined a one-unit system with $(n-1)$ warm standbys, where the operating unit is subject to repairable and nonrepairable failures and warm standbys are subject to only repairable failures. All distributions of times until failure as well as the distribution of repair times were assumed to be discrete phase type. A block replacement of the entire system occurs at the time when all units have experienced a nonrepairable failure and removed from the system. Tractable transitory and stationary characteristics of the system were determined.

The purpose of this paper is to extend the known analytic results for systems with warm standbys to the case that the systems under study are subject to both repairable and nonrepairable failures. Unlike the Ruiz-Castro and Fernandez-Villodre (2012) investigation, it will be assumed that nonrepairable failures occur to the system as a whole instead of a single operating unit. Such a perspective is useful for modeling a variety of practical scenarios; for example, the nonrepairable failure might represent the end of the system's useful life, perhaps resulting from product or process obsolescence, or the failure of a critical subsystem operating in series with the system composed of warm standbys. Alternatively, the exact duration of the mission of the system may not be known with certainty, and thus the occurrence of a nonrepairable failure could represent the end of the system's mission. In all of these cases, it is of interest to investigate the behavior of the system over a time period whose duration is described by a probability distribution. The interested reader is referred to the early work by Bryant and Murphy (1980) using this perspective applied to alternating renewal processes, and by Bryant and Wells (1984) using this perspective applied to more general systems. The occurrence of repairable and nonrepairable failures will also redirect our attention from first system failure and steady state or long run measures, such as availability, to measures which are more appropriate for systems viewed over finite but random time intervals.

1.1. Notation

I	Identity matrix
$\mathbf{0}, \mathbf{1}$	Vector of zeros, ones, respectively
X	Time until failure of an operating unit, $X \sim F(x) = 1 - e^{-\lambda x}$, $x \geqslant 0$, and $\lambda > 0$
W	Time until failure of the unit in standby, $W \sim F_1(w) = 1 - e^{-\lambda_1 w}$, $w \geqslant 0$, and $\lambda_1 > 0$
Y	Time required to repair a unit, $Y \sim G(y)$, $y \geqslant 0$
V	Time required to perform block replacement, $V \sim G_1(v)$, $v \geqslant 0$
T	Time until a nonrepairable failure, $T \sim H(t)$, $t \geqslant 0$
T_1	Time until the first system failure, $T_1 \sim K(t_1)$, $t_1 \geqslant 0$
U	Accumulated system uptime over the random interval $[0,T]$, $U \sim J(u)$, $u \geqslant 0$
$\tilde{f}(s)$	Scalar-valued Laplace-Stieltjes transformation of $F(x)$, $\tilde{f}(s) = \int_0^\infty e^{-st} dF(t)$
F_A	Matrix-valued Laplace Stieltjes transformation of $F(x)$, $F_A = \int_0^\infty \exp(At) f(t) dt$
Ψ	Random interval reliability, $P\{T_1 > T\}$
Λ	Random point availability, $P\{$System is operating at time $T\}$

The notational convention used throughout this paper is to use a capital letter to indicate a distribution function and the corresponding lower case letter for the density function if it exists. Thus $F(x)$ is the distribution function and $f(x) = \frac{dF}{dx}$ is the density function. The survival function is written as $\overline{F}(x) = 1 - F(x)$.

2. Preliminary concepts

2.1. Distributions of phase type

In the sequel, significant attention will be given to continuous distributions of phase type, or more simply, phase distributions. A distribution is of phase type if and only if it can be represented as the distribution of time until absorption in a finite state continuous-time Markov chain in which absorption is certain given any initial state (Neuts (1981)). Phase distributions are useful for representing evolutionary processes, such as the degradation of a component or the steps required to perform a repair, and are therefore appropriate for the current investigation. Moreover, phase distributions are dense in the class of life distributions and include all finite mixtures of convolutions of Erlang distributions; consequently, they are a robust assumption about the lifetime of a process under study. Finally, phase distributions can enhance computational tractability as indicated in the work of Perez-Ocon and Montoro-Cazorla (2006) and Ruiz-Castro and Fernandez-Villodre (2012).

Let A^* be the infinitesimal generator of a continuous-time Markov chain with $k+1$ states, $0, 1, 2, \ldots, k$, where state 0 is absorbing for any initial state. Let $\mathbf{0}$ be a $(1 \times k)$ vector of zeros and $\mathbf{1}$ a $(k \times 1)$ column of ones. A^* can be written as

$$A^* = \begin{pmatrix} 0 & \mathbf{0} \\ A_0 & A \end{pmatrix},$$

where A is a $k \times k$ stable matrix with negative diagonal elements and non-negative off-diagonal elements such that the row sums are nonpositive, and $A_0 = -A\mathbf{1}$. Let $\boldsymbol{\alpha}^*$ be the $1 \times (k+1)$ initial probability vector of the Markov chain. Then $\boldsymbol{\alpha}^* = [\alpha_0, \boldsymbol{\alpha}]$, with $\alpha_0 + \boldsymbol{\alpha}\mathbf{1} = 1$. For notational convenience, it is assumed throughout that $\alpha_0 = 0$.

If $H(t)$ is the distribution function associated with the time until absorption, then

$$H(t) = 1 - \boldsymbol{\alpha} \exp(At)\mathbf{1}, \quad t \geqslant 0, \tag{1}$$

and is absolutely continuous with probability density function

$$h(t) = -\boldsymbol{\alpha} A \exp(At)\mathbf{1}, \quad t \geqslant 0. \tag{2}$$

Since $\boldsymbol{\alpha}$ and A uniquely determine H, H is said to have representation $(\boldsymbol{\alpha}, A)$.

2.2. Matrix Laplace transformations

Because of the presence of phase distributions, matrix Laplace transformations will naturally arise in the calculations encountered in this work. A matrix Laplace transformation (MLT) is a $k \times k$ matrix defined by

$$F_A = \int_0^\infty \exp(At) f(t) dt \tag{3}$$

where $f(t)$ is a density function defined on $[0, \infty)$, and A is a $k \times k$ stable matrix as described in Section 2.1, (Bryant & Murphy,