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Original article

Exact solutions for nonlinear integro-partial differential equations using the generalized Kudryashov method

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A B S T R A C T

In this research, we construct the traveling wave solutions for some nonlinear evolution equations in mathematical physics. New solutions such as soliton solutions are found. The method used is the generalized Kudryashov method (GKM). We apply the method successfully to find the exact solutions of the following nonlinear integro-partial differential equations: the (1+1)-dimensional integro-differential Ito equation, (2+1)-dimensional integro-differential Sawada–Kotera equation and two members of integro-differential Kadomtsev–Petviashvili (KP) hierarchy equations. These equations have numerous important applications in mathematical physics as well as in engineering. This method is efficient, powerful and simple.

1. Introduction

Nonlinear partial differential equations (NPDEs) have a very important role in describing many nonlinear phenomena in chemistry, physics, mathematical biology, and many other fields of science and engineering. There are many methods for finding the exact solutions to nonlinear PDEs such as homogenous balance method [1,2], Darboux transform method [3,4], first integral method [5,6], tanh function method [7], modified simple equation method [8,9,10], auxiliary equations method [11,12], (G'/G)-expansion method [13,14], F-expansion method [15], Jacobi elliptic function method [16] and so on (see for example [17-26]). Recently Kudryashov [27] has presented a direct method namely truncated expansion method to discuss the analytic solutions for nonlinear evaluation equations. Kabir [28] has improved the truncated expansion method for nonlinear PDEs which called the improved of Kudryashov method. More recently Kaplan [29] has generalized the Kudryashov method to solve the nonlinear PDEs. In this paper, we have applied the generalized Kudryashov method to find the traveling wave solutions for the following nonlinear integro-partial differential equations:

(i) The $(1+1)$-dimensional integro-differential Ito equation [30]

$$u_{tt} + u_{xxxt} + 3(2u_x u_t + u u_{xt}) + 3u_{xx}\partial_x^{-1}(u_t) = 0\ . \tag{1}$$

(ii) The $(2+1)$-dimensional integro-differential Sawada–Kotera equation [31]

$$u_t = \left(u_{xxxx} + 5uu_{xx} + \frac{5}{3}u^3 + u_{xy}\right)_x - 5\partial_x^{-1}(u_{yy}) + 5uu_y + 5u_x\partial_x^{-1}(u_y). \tag{2}$$

(iii) The first integro-differential KP hierarchy equation [32–34]

$$u_t = \frac{1}{2}u_{xxy} + \frac{1}{2}\partial_x^{-2}[u_{yyy}] + 2u_x\partial_x^{-1}[u_y] + 4uu_y. \tag{3}$$

(iv) The second integro-differential KP hierarchy equation [32–34]

$$u_t = \frac{1}{16}u_{xxxxx} + \frac{5}{4}\partial_x^{-1}[uu_{yy}] + \frac{5}{4}\partial_x^{-1}\left[{u_y}^2\right] + \frac{5}{16}\partial_x^{-3}[u_{yyyy}] + \frac{5}{4}u_x\partial_x^{-2}[u_{yy}] + \frac{5}{2}u\partial_x^{-1}[u_{yy}] + \frac{5}{2}u_y\partial_x^{-1}[u_y] + \frac{15}{2}u^2\ u_x + \frac{5}{2}\ u_x\ u_{xx} + \frac{5}{4}u\ u_{xxx} + \frac{5}{8}\ u_{xyy}, \tag{4}$$

where ${\partial_x}^{-1} = \int_{-\infty}^{x} dx$.

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2. Description of the generalized Kudryashov method for NPDE

In this section, we consider the following nonlinear partial differential equation:-

$$P(v, v_t, v_x, v_{tt}, v_{xx}, v_{xt}, \ldots) = 0, \quad (5)$$

where P is a polynomial in the unknown function $v = v(x, t)$ and its partial derivatives.

The basic steps in the application of the GKM detailed in the following [29]:

Step 1. We assume that:-

$$v(x,t) = V(\xi), \ \xi = x - kt, \quad (6)$$

where k is an arbitrary constant. Eq. (6) leads to get:

$$P\left(V, V', V'', \ldots\right) = 0. \quad (7)$$

Step 2. We suppose the exact solution of Eq. (7) to be in the following rational form:-

$$V(\xi) = \frac{\sum_{i=0}^{N} a_i Q^i(\xi)}{\sum_{j=0}^{M} b_j Q^j(\xi)} = \frac{A[Q(\xi)]}{B[Q(\xi)]}, \quad (8)$$

where $a_i (i = 0, 1, 2, \ldots, N)$ and $b_j (j = 0, 1, 2, \ldots, M)$ are constants to be determined later such that $a_N \neq 0$, $b_M \neq 0$ and $Q = \frac{1}{1 \pm e^{\xi}}$. The function Q has then to satisfy the first order Bernoulli differential equation:

$$Q'(\xi) = Q^2(\xi) - Q(\xi). \quad (9)$$

Step 3. Determine the positive integer numbers N and M in Eq. (8) by balancing the highest order derivatives and the nonlinear terms in Eq. (7).

Step 4. Substituting Eqs. (8) and (9) into Eq. (7), we obtain a polynomial in Q^{i-j}, $(i, j = 0, 1, 2, \ldots)$. Setting all coefficients of this polynomial to be zero, we obtain a system of algebraic equations which can be solved by the Maple or Mathematica software package to get the unknown parameters $a_i (i = 0, 1, 2, \ldots, N)$ and $b_j (j = 0, 1, 2, \ldots, M)$. Consequently, we obtain the exact solutions of Eq. (5).

3. Applications of the generalized Kudryashov method for nonlinear integro-PDEs

Zhao et al. [30] have used the extended monoclinic test for study the exact solutions for Ito equations. Yu [31] have study the exact solution of integro-differential Sawada–Kotera by using the bilinear method. Gepreel [34] and Mohamed et al. [23] have studied the exact and approximate solutions to the integro-differential KP hierarchy equations by using (f/g) expansion method and reduced differential transform method respectively. In this section, we use the generalized Kudryashov method to find the traveling wave solutions for some nonlinear evolution equations. We shall namely solve the $(1+1)$-dimensional integro-differential Ito equation, the $(2+1)$-dimensional integro-differential Sawada–Kotera equation, and two members of the integro-differential KP hierarchy equations. These equations have very important applications in the area of mathematical physics.

3.1. The generalized Kudryashov method for the (1+1)-dimensional integro-differential Ito equation

In this subsection, we apply the generalized Kudryashov method to construct the exact solution to the $(1+1)$-dimensional integro-differential Ito Eq. (1). We use the transformation

$$u(x,t) = v_x(x,t). \quad (10)$$

This transformation changes the $(1+1)$-dimensional integro-differential Ito Eq. (1) to the following nonlinear partial differential equation:

$$v_{ttx} + v_{xxxxt} + 6v_{xx}v_{xt} + 3v_x v_{xxt} + 3v_{xxx}v_t = 0. \quad (11)$$

The traveling wave transformation (6) converts Eq. (11) to the following ODE:-

$$k^2V''' - kV^{(5)} - 6k\left(V''\right)^2 - 6kV'V''' = 0. \quad (12)$$

By integrating twice, we obtain:

$$kV' - V''' - 3\left(V'\right)^2 + C_1 = 0, \quad (13)$$

where $' = \frac{d}{d\xi}$ and C_1 is the integration constant. The relation between N and M in Eq. (8) is given by balancing the highest order V''' and the nonlinear term $(V')^2$ in (13), as follows:

$$N = M + 1. \quad (14)$$

Eq. (14) has an infinite number of solutions. For the special case in which we choose $M = 1$, we obtain:

$$V(\xi) = \frac{a_0 + a_1 Q + a_2 Q^2}{b_0 + b_1 Q}, \quad (15)$$

where a_0, a_1, a_2, b_0 and b_1 are constants to be determined later. Now substituting Eq. (15) into Eq. (13) and cleaning the denominator, we get a polynomial in $Q(\xi)$. Setting each power coefficient of $Q(\xi)$ to be zero, we get a system of nonlinear of algebraic equations. With the help of Maple, we solve the system of algebraic equation to get the following results:

Case 1.

$$C_1 = 0, \ a_0 = -\frac{1}{2}a_1, \ a_2 = 4b_0, \ b_1 = -2b_0, \ k = 4. \quad (16)$$

When, we substitute (16) into (15), we get:

$$V_1(\xi) = -\frac{1}{2}\left[\frac{-8b_0 - 2a_1\left(1 \pm e^{\xi}\right) + a_1\left(1 \pm e^{\xi}\right)^2}{b_0\left(1 \pm e^{\xi}\right)\left(-1 \pm e^{\xi}\right)}\right] \quad (17)$$

where $\xi = x - 4t$. Consequently, the solution of the $(1+1)$-dimensional integro-differential Ito equation takes the form:

$$u_1(x,t) = -\frac{8\left(\pm e^{[2x-8t]}\right)}{\left(1 \pm e^{[x-4t]}\right)^2\left(-1 \pm e^{[x-4t]}\right)^2}. \quad (18)$$

Case 2.

$$C_1 = 0, \ a_0 = \frac{b_0(a_1 + 2\ b_0)}{b_1}, \ a_2 = -2b_1, \ k = 1. \quad (19)$$

Eqs. (20) and (16) lead to get

$$V_2(\xi) = \frac{-2b_1 + (a_1 + 2b_0)\left(1 \pm e^{\xi}\right)}{b_1\left(1 \pm e^{\xi}\right)}, \quad (20)$$

where $\xi = x - t$. Hence the traveling wave solution for the $(1+1)$-dimensional integro-differential Ito equation takes the form:

$$u_2(x,t) = \frac{2\left(\pm e^{[x-t]}\right)}{\left(1 \pm e^{[x-t]}\right)^2}. \quad (21)$$

Case 3.

$$C_1 = 0, \ \ a_1 = \frac{a_0b_1 - 2{b_0}^2 - 2b_0b_1}{b_0}, \ \ a_2 = 0, \ \ k = 1. \quad (22)$$

Substitute (22) into (15), we have

$$V_3(\xi) = \frac{a_0b_1 - 2{b_0}^2 - 2b_0b_1 + a_0b_0\left(1 \pm e^{\xi}\right)}{b_0\left(b_0\left(1 \pm e^{\xi}\right) + b_1\right)}, \quad (23)$$

where $\xi = x - t$. Then, we deduce the exact solution of the $(1+1)$-dimensional integro-differential Ito equation as follows:

$$u_3(x,t) = \frac{2b_0(b_0 + b_1)\left(\pm e^{[x-t]}\right)}{\left(\pm b_0 e^{[x-t]} + b_0 + b_1\right)^2}. \quad (24)$$