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The corona between cycles and paths

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ABSTRACT

A graph is said to be cordial if it has a 0–1 labeling that satisfies certain properties. The corona $G_1 \odot G_2$ of two graphs G_1(with n_1 vertices and m_1 edges) and G_2 (with n_2 vertices and m_2 edges) is defined as the graph obtained by taking one copy of G_1 and n_1 copies of G_2, and then joining the ith vertex of G_1 with an edge to every vertex in the ith copy of G_2. In this paper we investigate the cordiality of the corona between cycles C_n and paths P_n, namely $C_n \odot P_m$.

1. Introduction

It is known that graph theory and its branches have become interest topics for almost all fields of mathematics and also other area of science such as chemistry, biology, physics, communication, economics, engineering, operations research, and especially computer science.

A graph labeling is an assignment of integers to the vertices or edges, or both, subject to certain conditions. There are many contributions and different kinds of labeling [1–9].

Two of the most important types of labeling are called graceful and harmonious. Graceful labeling was introduced independently by Rosa [10] in 1966 and Golomb [11] in 1972, while harmonious labeling were first studied by Graham and Sloane [12] in 1980. A third important type of labeling, which contains aspects of both of the other two, is called cordial and was introduced by Cahit [1] in 1990. Whereas the label of an edge vw for graceful and harmonious labeling is given respectively by $|f(v)-f(w)|$ and $(f(v)+f(w))$ (modulo the number of edges), cordial labeling use only labels 0 and 1 and the induced edge label $(f(v)+f(w))$(mod2), which of course equals $|f(v)-f(w)|$. Because arithmetic modulo 2 is an integral part of computer science, cordial labelings have close connections with that field. An excellent reference on this subject is the survey by Gallian [13].

More precisely, cordial graphs are defined as follows: Let $G=(V,E)$ be a graph, let f: $V \to \{0, 1\}$ be a labeling of its vertices, and let f^*: $E \to \{0, 1\}$ is the extension of f to the edges of G by the formula $f^*(vw)=(f(v)+f(w)) \pmod 2$. Thus, for any edge $e=uv$, $f^*(e)=0$ if its two vertices have the same label and $f^*(e)=1$ if they have different labels. Let v_0 and v_1 be the numbers of vertices labeled 0 and 1 respectively, and let e_0 and e_1 be the corresponding numbers of edges. Such a labeling is called *cordial* if both $|v_0-v_1| \le 1$ and $|e_0-e_1| \le 1$ hold. A graph is called *cordial* if it has a cordial labeling.

Suppose that $G=(V,E)$ is a graph, where V is the set of its vertices and E is the set of its edges. Throughout, it is assumed G is connected, finite, simple and undirected.

The *corona* $G_1 \odot G_2$ of two graphs G_1(with n_1 vertices and m_1 edges) and G_2 (with n_2 vertices and m_2 edges) is defined as the graph obtained by taking one copy of G_1 and n_1 copies of G_2, and then joining the ith vertex of G_1 with an edge to every vertex in the ith copy of G_2 [14]. It follows from the definition of the corona that $G_1 \odot G_2$ has $n_1+n_1n_2$ vertices and $m_1+n_1m_2+n_1n_2$ edges. It is easy to see that $G_1 \odot G_2$ is not in general isomorphic to $G_2 \odot G_1$. In [15], the corona of $P_n \odot C_m$ has been studied and proved that $P_n \odot C_m$ is cordial if and only if $(n, m) \ne (1, 3)$(mod4). In this paper we show that the corona $C_n \odot P_m$ is cordial for all $n \ge 3$ and $m \ge 1$.

2. Terminology and notation

A path with n vertices and $n-1$ edges is denoted by P_n, and a cycle with n vertices and n edges is denoted by C_n. Given a cycle or a path with $4r$ vertices, we let L_{4r} denote the labeling 0011 ... 0011 (repeated $r$times). In most cases, we then modify this by adding symbols at one end or the other (or both); thus $010L_{4r}$ denotes the labeling 010 0011 ... 0011 of the cycle

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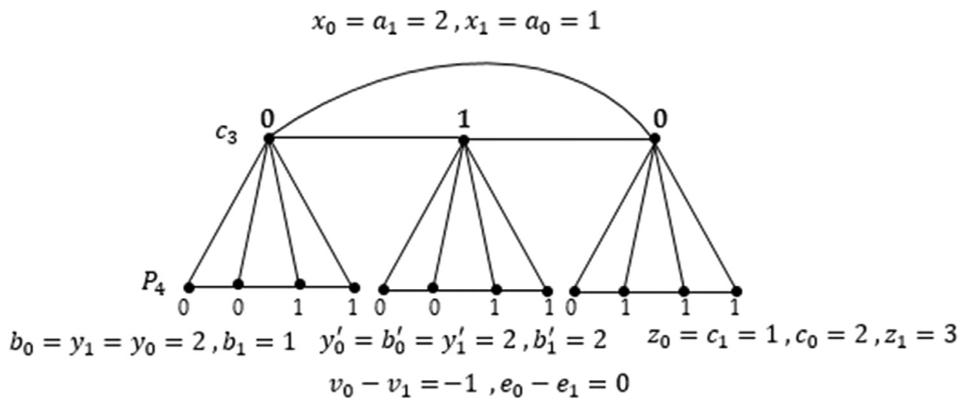

Fig. 1. .

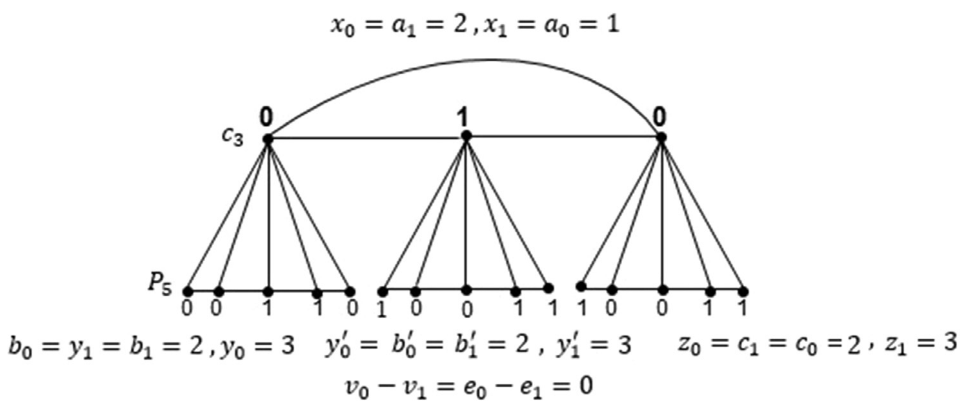

Fig. 2. .

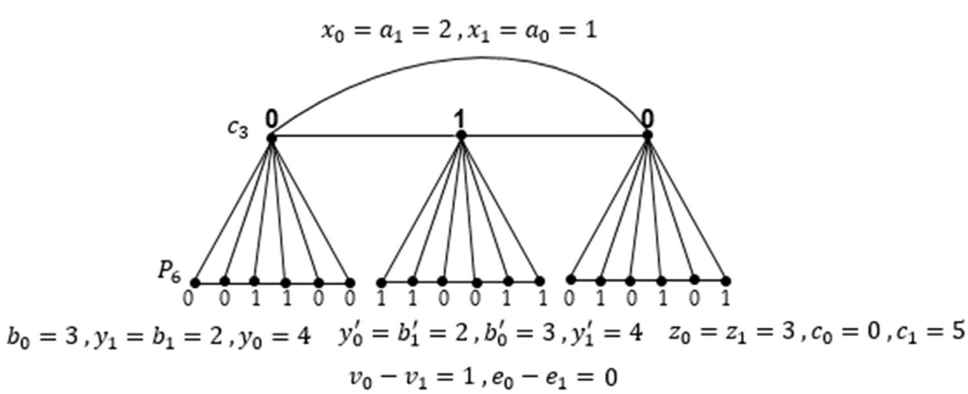

Fig. 3. .

C_{4r+3} (or the path P_{4r+3}) when $r \geq 1$ and 010 when $r = 0$. Similarly, $L_{4r}01$ is the labeling 0011 ... 0011 01 of the cycle C_{4r+2} (or the path P_{4r+2}) when $r \geq 1$ and 01 when $r = 0$, and so on. We write M_r for the labeling 01 ... 01 if r is even and 01 ... 010 if r is odd, for example, $M_6 = 010101$ and $M_7 = 0101010$. Also, we write 0_r for the labeling 0 ... 0 ($r$times) and 1_r for the labeling 1 ... 1 ($r$times) [3–8]. If G and H are two graphs, where G has n vertices, the labeling of the corona $G \odot H$ is often denoted by $[A : B_1, B_2, B_3, \ \ldots \ , B_n]$, where A is the labeling of the n vertices of G, and B_i, $1 \leq i \leq n$ is the labeling of the vertices of the copy of H that is connected to the ith vertex of G [2]. For a given labeling of the corona $G \odot H$, we denote v_i and e_i $(i = 0, 1)$ to represent the numbers of vertices and edges, respectively, labeled by i. Let us denote x_i and a_i to be the numbers of vertices and edges labeled by i for the graph G. Also, we let y_i and b_i be those for H, which are connected to the vertices labeled 0 of G. Likewise, let y'_i and b'_i be those for H, which are connected to the vertices labeled 1 of G. It is easy to verify that $v_0 = x_0 + x_0 y_0 + x_1 y'_0$, $v_1 = x_1 + x_0 y_1 + x_1 y'_1$, $e_0 = a_0 + x_0 b_0 + x_1 b'_0 + x_0 y_0 + x_1 y'_1$ and $e_1 = a_1 + x_0 b_1 + x_1 b'_1 + x_0(x_0 y_1) + x_1 y'_0$. Thus $v_0 - v_1 = (x_0 - x_1) + x_0(y_0 - y_1) + x_1(y'_0 - y'_1)$ and $e_0 - e_1 = (a_0 - a_1) + x_0(b_0 - b_1) + x_1(b'_0 - b'_1) + x_0(y_0 - y_1) - x_1(y'_0 - y'_1)$. In particular, if we have only one labeling for all copies of H, i.e., $y_i = y'_i$ and $b_i = b'_i$, then $v_0 = x_0 + n y_0$, $v_1 = x_1 + n y_1$, $e_0 = a_0 + n b_0 + x_0 y_0 + x_1 y_1$ and $e_1 = a_1 + n b_1 + x_0 y_1 + x_1 y_0$. Thus $v_0 - v_1 = (x_0 - x_1) + n(y_0 - y_1)$ and $e_0 - e_1 = (a_0 - a_1) + n(b_0 - b_1) + (x_0 - x_1)(y_0 - y_1)$, where n is the order of G.

3. Corona between cycles and paths

In this section, we show that the corona $C_n \odot P_m$ is cordial for all $n \geq 3$ and $m \geq 1$.

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