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Original Article

# Stochastic analysis of a duplicated standby system subject to shocks during repair

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ABSTRACT

Two probabilistic models of a duplicated standby system subjected to random shocks during the repair are discussed. Each unit consists of mixture between hardware and software components that work together and fail independently. The first model contains regular repairman. During the repair of hardware components the regular repairman can cause a shock and damage the unit. In the second model there is an expert repairman to avoid the occurrence of shocks and to study the effect of experience level on the system. Several reliability measures for the proposed system are obtained. Finally numerical study is done to clarify the results.



## 1. Introduction

With tremendous progress in the industrial field, several researchers studied the probabilistic behavior of many reliability models to calculate many parameters in the reliability field. But, no attention was directed to the effect of the shocks which may be result from the repairman during the process of repairing. [1,2] studied the effect of shock on the system and divided the source of shocks into two main parts, internal factors and external source. For example stress, strain, power failure, etc. [3] studied random shocks that affect the system without reference to the reasons for their occurrence. In addition, there are many systems which contain software and hardware subsystems. [4–8] performed some studies in this area, but they are not be exposed to software subsystems. [9] developed a combined reliability model for the system that contain hardware subsystems and software subsystems. [10] discussed the probabilistic analysis of a computer system and suggested that with preventive maintenance and by giving priority to software replacement, the system will be more effective. [11] formulated reliability model of a computer system and pointed out that, in order to raise the efficiency of the system must saving hardware redundancy in cold standby. [12] concluded that by increasing the hardware repair rate and by using preventive maintenance, the system model can be more efficient and beneficial to use. This paper themes to analyze a duplicated cold standby system, each unit contains both hardware and software components that work together and fail independently. There are two models, the first contains a regular repairman who visits the system immediately when required and in case of failure due to hardware failure the repairman can cause a shock during the repair. This causing damage of the unit so it will be replaced. For example during the repair of the direct digital control panel (DDC-Panel) that control fan coil unit (FCU), the repairman may cause any short circuit or wrong connection and damage the digital output of the control module. When the failure occurred due to software failure the repairman will repair it. By replacing the regular repairman with another one which is called expert repairman as shown in the second model, the occurrence of shocks will be avoided.

The following measures of system reliability can be derived: The mean time to system failure (MTSF), Steady-state availability (SSA), Steady-state busy period due to (hardware, software and replacement) and profit analysis in the steady-state.

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## 2. General assumptions

1- There are 2 similar units in the system.
2- Initially one of them is operating and the other is in cold standby state.
3- The online unit faces software and hardware failures.
4- There is one repairman and always available.
5- After software failure, the repair of the unit can be performed by the repairman.
6- Failure, repair, replacement and shock times follow exponential distributions with different rates.
7- The connected switch works perfectly and instantaneously.
8- After completion of repair, the unit is as good as new.
9- The system is totally down when all its units are failed.

### *2.1. Assumptions for model I*

1- In case of hardware failure, the regular repairman goes to repair the unit and can cause a shock during the repair of hardware failure.
2- If the regular repairman caused a shock during the repair of hardware failure then the unit will be replaced by a new one.

### *2.2. Assumptions for model II*

1- The repairman is called an expert repairman.
2- After hardware failure, the repair of the unit can be performed by the repairman without making any shock.

## 3. Notations

| | |
|---|---|
| $\eta_1$: | constant hardware failure rate. |
| $\varepsilon_1$: | constant software failure rate. |
| $\tau$: | constant shock rate. |
| $\eta_2$: | constant hardware repair rate. |
| $\varepsilon_2$: | constant software repair rate. |
| $\psi$: | constant replacement rate. |
| *O*: | the unit is operating. |
| *CS*: | the unit is in cold standby mode. |
| *rh*: | the repair of hardware failed unit. |
| *rs*: | the repair of software failed unit. |
| *wrh*: | the hardware failed unit is waiting for repair. |
| *wrs*: | the software failed unit is waiting for repair. |
| *repl*: | unit under replacement. |
| $T_j(t)$: | *Pr* [the system is in state $j$ at instant $t \geq 0$, $j = 0, 1, \cdots, n$.] |
| $A(t)$: | *Pr* [the system is working at instant $t$]. |
| $Bh(t)$: | busy period due to hardware failure. |
| $Bs(t)$: | busy period due to software failure. |
| $Br(t)$: | busy period due to replacement. |
| $LFDE_s$: | linear first order differential equations. |
| $[\Phi_{ij}]$: | a matrix form of order $10 \times 10$. |
| $[\theta_{mn}]$: | a matrix form of order $4 \times 4$. |
| $[D_{hk}]$: | a matrix form of order $4 \times 1$. |
| PF: | the incurred profit of the system in the steady state. |
| SSBP: | steady state busy periods. |
| EPF: | expected total profit incurred to the system in the steady-state. |
| ○: | Working State. |
| □ : | Completely Failed State. |

$$T_j = \lim_{t\to\infty} T_j(t), \quad SSA = \lim_{t\to\infty} A(t), \quad SSBh = \lim_{t\to\infty} Bh(t),$$
$$SSBs = \lim_{t\to\infty} Bs(t), \; SSBr = \lim_{t\to\infty} Br(t)$$

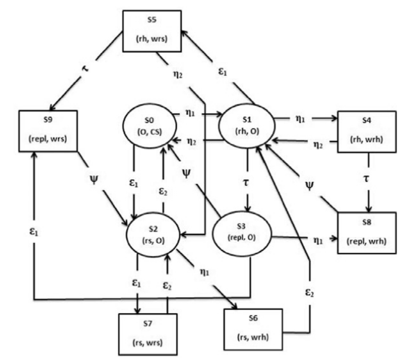


**Fig. 1.** Transition diagram of model I.

## 4. Model I

Fig. 1 shows the transition diagram of model I.
It is easy to verify that:

$$T(0) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \tag{1}$$

Based on the method of $LFDE_s$ for model I the following can be obtained.

$$\left.\begin{aligned}
T_0'(t) &= -(\varepsilon_1 + \eta_1)T_0(t) + \eta_2 T_1(t) + \varepsilon_2 T_2(t) + \psi T_3(t),\\
T_1'(t) &= -(\varepsilon_1 + \eta_1 + \eta_2 + \tau)T_1(t) + \eta_1 T_0(t) + \eta_2 T_4(t) +\\
&\quad \varepsilon_2 T_6(t) + \psi T_8(t),\\
T_2'(t) &= -(\varepsilon_1 + \eta_1 + \varepsilon_2)T_2(t) + \varepsilon_1 T_0(t) + \eta_2 T_5(t) +\\
&\quad \varepsilon_2 T_7(t) + \psi T_9(t),\\
T_3'(t) &= -(\varepsilon_1 + \eta_1 + \psi)T_3(t) + \tau T_1(t),\\
T_4'(t) &= -(\eta_2 + \tau)T_4(t) + \eta_1 T_1(t),\\
T_5'(t) &= -(\eta_2 + \tau)T_5(t) + \varepsilon_1 T_1(t),\\
T_6'(t) &= -\varepsilon_2 T_6(t) + \eta_1 T_2(t),\\
T_7'(t) &= -\varepsilon_2 T_7(t) + \varepsilon_1 T_2(t),\\
T_8'(t) &= -\psi T_8(t) + \tau T_4(t) + \eta_1 T_3(t),\\
T_9'(t) &= -\psi T_9(t) + \tau T_5(t) + \varepsilon_1 T_3(t).
\end{aligned}\right\} \tag{2}$$

Eq. (2) can be put in the following matrix form:

$$T'(t) = \zeta \times T(t). \tag{3}$$

where,

$$\left(T'(t)\right)^T = \left[\; T_0'(t)\; T_1'(t)\; T_2'(t)\; T_3'(t)\; T_4'(t)\; T_5'(t)\; T_6'(t)\; T_7'(t)\; T_8'(t)\; T_9'(t)\;\right],$$

$$(T(t))^T = [T_0(t)\; T_1(t)\; T_2(t)\; T_3(t)\; T_4(t)\; T_5(t)\; T_6(t)\; T_7(t)\; T_8(t)\; T_9(t)],$$

and

$$\zeta = [\Phi_{ij}], \tag{4}$$

where,

$$\begin{aligned}
\Phi_{11} &= -(\varepsilon_1 + \eta_1), \quad \Phi_{22} = -(\varepsilon_1 + \eta_1 + \eta_2 + \tau), \quad \Phi_{33} = -(\varepsilon_1 + \eta_1 + \varepsilon_2),\\
\Phi_{55} &= \Phi_{66} = -(\tau + \eta_2), \quad \Phi_{44} = -(\varepsilon_1 + \eta_1 + \psi), \quad \Phi_{13} = \Phi_{27} = \Phi_{38} = \varepsilon_2,\\
\Phi_{36} &= \Phi_{25} = \Phi_{12} = \eta_2, \quad \Phi_{42} = \Phi_{10,6} = \Phi_{95} = \tau, \quad \Phi_{73} = \Phi_{21} = \Phi_{52} = \Phi_{94} = \eta_1,\\
\Phi_{31} &= \Phi_{10,4} = \Phi_{83} = \Phi_{62} = \varepsilon_1, \quad \Phi_{3,10} = -\Phi_{10,10} = \psi, \quad \Phi_{14} = \Phi_{29} = -\Phi_{99} = \psi\\
-\Phi_{77} &= -\Phi_{81} = \varepsilon_2.
\end{aligned}$$

All other elements equal to 0.
The first part in Eq. (2) can be deduced from the following:

$$T_0(t + \Delta t) = [1 - (\varepsilon_1 + \eta_1)\Delta t]T_0(t) + \eta_2 T_1(t)\Delta t + \varepsilon_2 T_2(t)\Delta t + \psi T_3(t)\Delta t,$$