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Cilia walls influence on peristaltically induced motion of magneto-fluid through a porous medium at moderate Reynolds number: Numerical study

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A B S T R A C T

This article addresses, effects of a magneto-fluid through a Darcy flow model with oscillatory wavy walled whose inner surface is ciliated. The equations that governing the flow are modeled without using any approximations. Adomian Decomposition Method (ADM) is used to evaluate the solution of our system of nonlinear partial differential equations. Stream function, velocity and pressure gradient components are obtained by using the vorticity formula. The effects for our arbitrary physical parameters on flow characteristics are analyzed by plotting diagrams and discussed in details. With the help of stream lines the trapping mechanism has also been discussed. The major outcomes for the ciliated channel walls are: The axial velocity is higher without a ciliated walls than that for a ciliated walls and an opposite behaviour is shown near the ciliated channel walls. The pressure gradients in both directions are higher for a ciliated channel walls. More numbers of the trapped bolus in the absent of the eccentricity of the cilia elliptic path.

1. Introduction

Oscillatory wavy walled (Peristalsis) play an important phenomena in the transport of biofluids and involved in many biomechanical systems. Also, pumping phenomena with peristalsis is used for pumping corrosive materials so as to prevent direct contact of the fluid with the pump's internal surfaces. Moreover, by using the principle of peristalsis, some bio-mechanical instruments, as heart-lung machine have been simulated. Numbers of analytical [1–12], numerical and experimental [13–17] studies of peristaltic flows for different fluids have been discussed.

The effect of magnetic field and Darcy medium on the peristaltic mechanisms are important in connection with specific problems of the movements of the conductive physiological fluids, see [18–25].

Cilia, hair resembles animated appendages, presented in the respiratory, digestive and reproductive system of males and females and the nervous system in all categories of the animal kingdom. Cilia motion plays an important role in physiological processes such as locomotion, feeding, circulation, breathing, and reproduction. The cilia beatings, being well coordinated, generate a metachronal wave. The envelope of cilia tips, forming the metachronal wave, acts as an extensible wall with a tendency of forward motion always in the same direction. Some various studies about a cilia transport have been achieved by references [26–36].

No attempt has been made yet to study and solve this model by using the ADM method without any wavy walls approximations. Therefore in this paper, we will present the influence of the magnetic field and Darcy medium on peristaltic flow due to the cilia motion. The governing non-linear partial differential equations of this model have been solved with the ADM method. This method has already been used for the solutions of several other problems [43–46]. The impact of all the physical parameters is discussed and plotted. The paper is organized as follows. Section 2 includes the explanation of ADM briefly. The physical modeling statement with geometry of our problem is presented in Section 3. The Adomian Decomposition Method for solving our system equations is given in Section 4. In Section 5, we discussed in details the results we have obtained and the effects for our arbitrary physical parameters on the flow characteristics. Major findings of our model are included in the last section.

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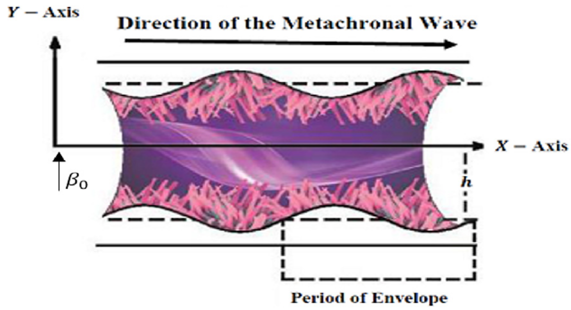

Fig. 1. Geometry of the problem.

2. The adomian decomposition method(ADM)

To explain this method [37–42] , we consider the following differential equation

$$Lu + Ru + Nu = g(x), \tag{1}$$

with prescribed conditions. In the above equation $u(x)$ is unknown scalar function, L is the highest-order derivative which is assumed to be easily invertible, R is a linear differential operator of order less than L, Nu represents the nonlinear terms, and g is an inhomogeneous term. Applying the inverse operator L^{-1} to both sides of Eq. (1), and using the given conditions we have

$$u(x) = f(x) - L^{-1}(Ru) - L^{-1}(Nu), \tag{2}$$

in which the function $f(x)$ represents the terms arising from the integration of $g(x)$ and then using prescribed intial or boundary conditions. For examble, if $L = \frac{d^3}{dx^3}$, then L^{-1} is a three-fold integration, and

$$f(x) = u(0) + xu'(0) + \frac{x^2}{2!}u'''(0) + L^{-1}g$$

.

The linear term $u(x)$ in terms of an infinite sum of components u_m is decomposed through the following equation

$$u(x) = \sum_{m=0}^{\infty} u_m(x). \tag{3}$$

The nonlinear operator Nu can be decomposed into a infinite series of polynomials as

$$Nu = \sum_{m=0}^{\infty} A_m(x). \tag{4}$$

The components of $u(x)$ can be determined recursively. $A_m(x)$ are the Adomian polynomials of u_0, u_1, $u_2 \cdots$, u_m and satify

$$A_m = \frac{1}{m!}\frac{d^m}{d\lambda^m}\left[N\left(\sum_{i=0}^{\infty}\lambda^i u_i\right)\right]_{\lambda=0}, \qquad m = 0, 1, 2\cdots \tag{5}$$

From Eqs. (2)-(4) we have

$$\sum_{m=0}^{\infty} u_m(x) = f(x) - L^{-1}\left(R\sum_{m=0}^{\infty} u_m\right) - L^{-1}\left(R\sum_{m=0}^{\infty} A_m\right). \tag{6}$$

The above equation easily gives

$$\begin{aligned} u_0 &= f(x), \\ u_{m+1} &= L^{-1}(Ru_m) - L^{-1}(A_m), \qquad m \geq 0. \end{aligned} \tag{7}$$

All components are determinable since A_0 depends only on u_0, A_1 depends on u_0, u_1, etc. Moreover, since the series is commonly rapidly convergent, the m-term partial sum $\phi_m = \sum_{i=0}^{m-1} u_i$ could be the practical solution.

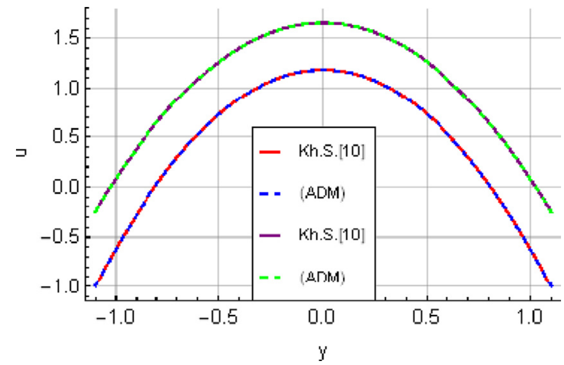

Fig. 2. Comparison between present method($\alpha \neq 0$, $R_e \neq 0$) and result obtained by Mekheimer [10]. (a) $\phi = 0.1$, $Q = 0.5$; (b) $\phi = 0.3$, $Q = 1$.

Table 1
Comparison of longitude velocity distribution for present work when $\alpha = 0.01$, $R_e = 0.01$ and work obtained by Mekheimer [10] for fixed values of $\phi = 0.1$, $q = 0.5$.

y	u(x,y) Present work	u(x,y) Mekheimer [10] with long wavelength approximation	|error|
−1.1	−1	−1	0
−0.9	−0.278794	−0.278738	−0.0000566376
−0.7	0.298227	0.298272	−0.0000445125
−0.5	0.731027	0.731029	$-1.91459*10^{-6}$
−0.3	1.01958	1.01953	0.0000424383
−0.1	1.16386	1.16379	0.0000694011
0	1.18189	1.18182	0.0000729958

Regarding the convergence of the decomposition method and the detailed description of the Adomian decomposition and the modified decomposition algorithms, we refer the readers to the studies [43–46].

3. Statement of physical model

Let's take in our consideration an incompressible MHD flow with a Darcy flow model in symmetric channel. Flow occurs due to the metachronal wave which is produced due to collective beating of the cilia with constant speed c along the walls of the channel whose inner surface is ciliated (Fig. 1). The plates of the channel are assumed to be electrically insulated. We assume that a uniform magnetic field strength β_0 is applied in the transverse direction to the direction of the flow and the induced magnetic field is assumed to be negligible. The geometry of the envelope of cilia tips and the flow of fluid are represented in two coordinate systems; one, fixed in the space, is (X, Y) and another is (x', y') moving to the right with a speed c relative to the fixed one. The two coordinate systems are connected with the relation.

$$X = x' + ct, \qquad Y = y', \qquad U = u' + c, \qquad V = v', \tag{8}$$

where (U, V) and (u', v') are components of velocity in the fixed and moving systems respectively.

The channel walls surface (in fixed systems) are given by (Fig. 1):

$$H(X,t) = \pm\left[h + \epsilon\cos\left[\frac{2\pi}{\lambda}(X - ct)\right]\right], \tag{9}$$

or (in moving systems) written as:

$$\eta'(x') = \pm\left[h + \epsilon\cos\left[\frac{2\pi}{\lambda}x'\right]\right], \tag{10}$$

where, t is time, λ is wavelength, ϵ is the wave amplitude, h is mean distance of the wall from the central axis and c is wave velocity of the metachronal wave. Following Sleigh [26], the cilia tips