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Stable project allocation under distributional constraints[☆]

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ABSTRACT

In a two-sided matching market when agents on both sides have preferences the stability of the solution is typically the most important requirement. However, we may also face some distributional constraints with regard to the minimum number of assignees or the distribution of the assignees according to their types. These two requirements can be challenging to reconcile in practice. In this paper we describe two real applications, a project allocation problem and a workshop assignment problem, both involving some distributional constraints. We used integer programming techniques to find reasonably good solutions with regard to the stability and the distributional constraints. Our approach can be useful in a variety of different applications, such as resident allocation with lower quotas, controlled school choice or college admissions with affirmative action.

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1. Introduction

A centralised matching scheme has been used since 1952 in the US to allocate junior doctors to hospitals [40]. Later, the same technology has been used in school choice programs in large cities, such as New York [3] and Boston [4]. Similar schemes have been established in Europe for university admissions and school choice as well. For instance, in Hungary both the secondary school and the higher education admission schemes are organised nationwide, see [12] and [13], respectively. Furthermore, it can also be used to allocate courses to students under priorities [20]. In the above mentioned applications it is common that the preferences of the applicants and the rankings of the parties on the other side are collected by a central coordinator and a so-called stable allocation is computed based on the matching algorithm of Gale and Shapley [26]. Two-sided matching markets, and the above applications in particular, have been extensively studied in the last decades, see [43] and [35] for overviews from game theoretical and computational aspects, respectively.

In this paper we describe two recent applications at the Corvinus University of Budapest, where we used a similar method with some interesting caveats. In the first application we had to allocate students to projects in such a way that the number of students allocated to each project is between a lower and an upper quota, together with an additional requirement over the distribution of the foreign students. This is a natural requirement present in many applications, such as the Japanese resident allocation scheme [30]. In the second application we scheduled students to companies for solving case studies in a conference, and here again we faced some distributional constraints.

We decided to use integer programming techniques for solving both applications. We had at least three reasons for choosing this technique. The first is that with IP formulations we can easily encode those distributional requirements that the organisers requested, so this solution method is robust to accommodate special features. The second reason is that the computational problem became NP-hard as the companies submitted lists with ties. Using ties in the ranking was by our recommendation to the companies, because ties give us more flexibility when finding a stable solution under the distributional constraints. We describe this issue more in detail shortly. Finally, our third reason for choosing IP techniques was that it facilitates multi-objective optimisation, e.g. finding a

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most-stable solution if a stable solution does not exist under the strict distributional constraints.

The usage of integer programming techniques for solving two-sided stable matching problems is very rare in the applications, and the theoretical studies on this topic have only started very recently. The reason is that the problems are relatively large in most applications, and the Gale-Shapley type heuristics are usually able to find stable solutions, even in potentially challenging cases. A classical example is the resident allocation problem with couples, which has been present in the US application for decades, and it is still solved by the Roth-Peranson heuristic [42]. The underlying matching problem is NP-hard [39], but heuristic solutions are quite successful in practice, see also [14] on the Scottish application. However, integer programming and constraint programming techniques have been developed very recently and they turned out to be powerful enough to solve large random instances [15,18,21]. Similarly encouraging results have been obtained for some special college admission problems, which are present in the Hungarian higher education system. These special features also make the problem NP-hard in general, but at least one of these challenging features, turned out to be solvable even with real data involving more than 150,000 applicants [6]. Finally, the last paper that we highlight with regard to this topic deals with the problem of finding stable solutions in the presence of ties [34]. However, we are not aware of any papers that would study IP techniques for the problem of distributional constraints.

Distributional constraints are present in many two-sided matching markets. In the Japanese resident allocation the government wants to ensure that the doctors are evenly distributed across the country, and to achieve this they imposed lower quotas on the number of doctors allocated in each region [27,30–32]. Distributional objectives can also appear in school choice programs, where the decision makers want to control the socio-ethnical distribution of the students [2,17,22,23,33]. Nguyen and Vohra [37] studied a special case where soft constraints are imposed on the proportion of different types of students. Furthermore, the same kind of requirements are implemented in college admission schemes with affirmative action [1] such as the Brazilian college admission system [8] and the admission scheme to Indian engineering schools [9].

Finally, there is a recent line of research by mathematicians on so-called classified stable matchings, where the problem of finding a stable solution under lower and upper quotas over certain types of applicants. Huang [28] gave an efficient algorithm for laminar set systems, which was generalised by Fleiner and Kamada [25] is a matroid framework, and further extended by Yokoi [44] for poly-matroids. Finally, Yokoi provided an efficient method for finding an envy-free matching for so-called paramodular lower and upper quota functions, if such a solution exists, and she also proves that the problem is NP-hard in the general setting. A model with one-sided preferences and ditributional constraints was studied recently in [7].

When stable solutions do not exist for the strict distributional constraints then we either need to relax stability or to adjust the distributional constraints. In this study we will consider the trade-off between these two goals, and develop some reasonable solution concepts.

Here, we briefly describe our definitions and solution concepts, the precise formulations will follow as we develop our model and solution concepts under extending sets of constraints. In our model the applicants submit their strict preferences on the companies and the companies provide weak rankings over the applicants. The companies have lower and upper quotas respecting the number of assignees. A matching is feasible if it respects these quotas. A matching is stable if for any applicant-company pair not in the matching either the applicant prefers her matching or the company has filled its upper quota with weakly higher ranked applicants. A matching is envy-free if no applicant has a justified envy towards another applicant, meaning that she prefers the company where the other applicant is admitted to her assignment and she is also ranked strictly higher by that company than the other applicant. An envy-free matching may be wasteful, meaning that there can be unfilled companies that are preferred by some applicants to their assignments. A matching is stable if and only if it is envy-free and non-wasteful. When the applicants have types then we may also have lower and upper quotas with respect to the types, which have to be obeyed for the feasibility of the matching. These quotas may apply for individual companies (as in our first application), for sets of companies, or for all companies (as in our second application). In our model (and motivating applications) the applicants are partitioned according to their types (such as domestic and foreign students). A matching is within-type envy-free if there is no justified envy between any two students of the same type.

Regarding the solution concepts, we are focusing on “almost stability”. A stable matching may not exist when both lower and upper quotas are imposed. In this case a natural solution is to look for an envy-free matching, which is as non-wasteful as possible. If envy-free matching does not exist either, then we may want to find a feasible matching where the number of pairs with justified envy is minimised. If the applicants have types and an envy-free matching does not exist, then we can look for within-type envy-free matchings. This solution is guaranteed to exist under some natural assumptions, which are satisfied in our applications (Theorem 1). We can also characterise these matchings by the usage of type-specific scores, where the applicants of certain types can get extra scores (Theorem 2). Finally, among the within-type envy-free matchings we may want to minimise envy across types, i.e. minimise the pairs of applicants with different types that have justified envies. In this minimisation we can simply take the number of such pairs, or alternatively we can consider the intensity of the envy (how much higher the rejected applicant is compared to an unfairly accepted applicant) and we may aim to minimise the total intensity of the envies.

We developed integer programming formulations to solve these problems arising from two real applications, and we report the solutions that we obtained in our case studies.

2. Definitions and preliminaries

Many-to-one stable matching markets have been defined in many contexts in the literature. In the classical college admissions problem by Gale and Shapley [26] the students are matched to colleges. In the computer science literature this problem setting is typically called Hospital / Residents problem (HR), due to the National Resident Matching Program (NRMP) and other related applications. In our paper we will refer the two sets as *applicants* $A = \{a_1, \dots, a_n\}$ and *companies* $C = \{c_1, \dots c_m\}$. Let u_j denote the upper quota of company c_j.

Regarding the preferences, we assume that the applicants provide strict rankings over the companies, but the companies may have ties in their rankings. The preference lists of the applicants may be incomplete in our model (so not all the applicant-company pair is possible), but in our applications the preference lists are complete, and this condition is also used in some of our theoretical results. This model is sometimes referred to as Hospital / Residents problem with Ties (HRT) in the computer science literature, see e.g. [35]. In our context, let r_{ij} denote the rank of company c_j in a_i's preference list, meaning that applicant a_i prefers c_j to c_k if and only if $r_{ij} < r_{ik}$. Let s_{ij} be an integer representing the score of a_i by company c_j, meaning that a_i is preferred over a_k by company c_j if $s_{ij} > s_{kj}$. Note that here two applicants may have the same score at a company, so $s_{ij} = s_{kj}$ is possible. Let $\bar{s}$ denote the maximum

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