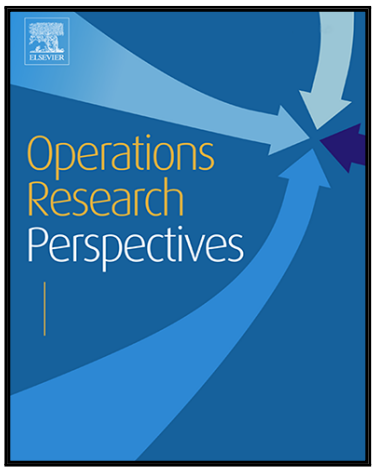

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Abstract. We study the existence and uniqueness of equilibria for perfectly competitive markets in capacitated transport networks. The model under consideration is rather general so that it captures basic aspects of related models in, e.g., gas or electricity networks. We formulate the market equilibrium model as a mixed complementarity problem and show the equivalence to a welfare maximization problem. Using the latter we prove uniqueness of the resulting equilibrium for piecewise linear and symmetric transport costs under additional mild assumptions. Moreover, we show the necessity of these assumptions by illustrating examples that possess multiple solutions if our assumptions are violated.

1. Introduction

We consider perfectly competitive markets upon capacitated networks, where transport costs are modeled using piecewise linear and symmetric cost functions. In this setting, we prove uniqueness of market equilibria under mild assumptions. Our motivation is the following. On the one hand, uniqueness of market equilibria is a classical topic of mathematical economics by itself. On the other hand, our model including a networked transport infrastructure has important applications in the areas of, e.g., electricity and gas markets. Furthermore, uniqueness of equilibria of such models is an important prerequisite for studying more complicated, e.g., multilevel, market models; see, e.g., Daxhelet and Smeers (2001), Grimm, Grübel, et al. (2017), Grimm, Kleinert, et al. (2017), Grimm, Martin, et al. (2016), Hobbs, Metzler, et al. (2000), Hu and Ralph (2007), and Kleinert and Schmidt (2018) for multilevel models in electricity markets as well Grimm, Grübel, et al. (2017) and Grimm et al. (2017a) for multilevel models of gas markets. Most of the above mentioned papers abstract from transport costs. However, there also exist equilibrium models including transport costs on networks. These are mainly studied in the context of imperfect competition in gas markets, cf., e.g., Cremer, Gasmi, et al. (2003) and Cremer and Laffont (2002). For an application in the electricity sector, see Paz (2015) or Hobbs and Rijkers (2004) for a more general study. In addition to the fields of gas and electricity, one might also think of other networked transport structures like they appear in water or traffic networks. However, these fields are somehow different. In traffic networks the classical concept of equilibria is the Wardrop equilibrium, cf. Wardrop (1952) and Wardrop and Whitehead (1952), which differs from the equilibria yielding market clearing prices in our context. Finally, in the context of water networks, the techno-economic literature focuses on different issues like market power and institutional constraints due to the complex nature of water rights; cf., e.g., Chakravorty et al. (2009).

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