



# Single machine scheduling with two-agent for total weighted completion time objectives

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## ABSTRACT

This paper considers two-agent scheduling problem with a single machine which is responsible for processing jobs from two agents. The objective is to minimize the objective function of one agent, subject to an upper bound on the objective function of the other agent. The objectives considered in this paper are, (1) the minimization of total completion time and (2) the minimization of total weighted completion time. To solve these problems, one heuristic and an Ant Colony Optimization algorithm are proposed. The heuristic suggested in the paper are motivated by the Weighted Shortest Processing Time first (WSPT) rule. A numerical experiment is performed on randomly generated problem instances. The performance of the algorithm is evaluated by comparing it with the lower bound value of all three problems considered in the present paper.

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## 1. Introduction

Scheduling problems for optimizing resource allocation have been studied for many years, as scheduling plays a critical role in many manufacturing and service industries. Furthermore, in manufacturing and service industries, the scheduling decision can have a significant impact on improving the productivity and efficiency of the production system.

During last two decades, two-agent scheduling problems have been used in many systems, such as railroad allocation systems [1], aircraft landing systems [2], telecommunication systems [3], and cloud computing systems [4] etc. Baker and Smith [5] first considered the two-agent scheduling problem. In this problem, two agents compete for limited production resources. Each agent has a set of jobs to be processed using a common processing source. Also, each agent may have different objectives at a given time. For instance, the objective functions could be the minimization of weighted tardiness and earliness [6–8], the minimization of total completion time [9–11] or the minimization of the number of tardiness jobs [12,13].

The two-agent scheduling problem faces the dilemma of minimizing two conflicting objectives associated with two agents. In the present paper, the objectives of two-agents are in conflict, because the completion time of jobs for the two agents are mutually inde-

pendent. To address the conflicting nature of objective functions, Suresh and Chaudhuri [14] suggested two approaches: (1) assign weight to the objective of each agent and minimize the weighted objective function; (2) minimize the objective function of one agent while keeping the objective function of the other agent within a pre-specified level. T. E. Cheng et al. [15] named these two types of single machine with multi-agent scheduling problem as a “minimality model” and a “feasibility model,” respectively. The problems studied in the present paper is the feasibility model, in which the objective function of one agent is minimized, while keeping the objective function of the other agent within a pre-specified limit.

In the present paper, we study three two-agent scheduling problems with a total weighted completion time and total completion time objectives. In these problems, all jobs from two agents are processed by a common machine. For each problem, the processing time and weight of each job is given. The problem involves finding a schedule that minimizes the objective function of one agent, subject to an upper bound on the objective function of the other agent. In this paper, depending upon the combination of objectives, three problems are considered.

**Problem 1.** The objective here is to minimize the total weighted completion time of the one agent, subject to an upper bound on the total weighted completion time of the other agent.

**Problem 2.** The objective here is to minimize the total weighted completion time of the one agent, subject to an upper bound on the total completion time of the other agent.

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**Problem 3.** The objective here is to minimize the total completion time of the one agent, subject to an upper bound on the total weighted completion time of the other agent.

The novelty of this paper is to introduce three new problems, develop a heuristic and a meta-heuristic, and introduce some benchmark problem instances for future research. The proposed ant-colony meta-heuristic solves the problem in two phases and accommodates the specific requirement of the considered problems.

The structure of the paper is as follows. Section 2 reviews the relevant literature and provides a general review of previous works. Section 3 offers a problem description and mathematical formulations of the two-agent scheduling problems. In Section 4, we present the proposed heuristic algorithm used for solving the scheduling problems studied in the present paper. In Section 5, a numerical experiment is preformed, and the corresponding results are reported. Section 6 provides a conclusion to this paper.

## 2. Literature review

The single machine with two agents scheduling problem was introduced by Baker and Smith [5]. They examined three basic objective functions: minimizing makespan, maximum tardiness, and weighted completion time. The various problems studied by Baker and Cole Smith [16] are considered to be NP-hard, as proved by Allesandro Agnetis et al. [9,17].

There were a number of papers published after the work of Baker and Cole Smith [16] and Allesandro Agnetis et al. [9]. These papers studied various two-agent scheduling problems from the perspectives of computational complexity and algorithms. Wu et al. [18] studied a single-machine two-agent scheduling problem with release times and developed algorithms to minimize the total completion time for both agents. Oron et al. [19] studied various single-machine two-agent scheduling problems with equal job processing times, where the objective was to minimize the weighted total completion time and weighted number of tardy jobs. Ng et al. [20] investigated the complexity of a two agents scheduling problem on a single machine, where the objective was to minimize the total completion time of the first agent, subject to an upper bound on the number of tardy jobs of the second agent. Alessandro Agnetis et al. [17] studied different two-agent scheduling problems with the objective function of a total completion time, a total weighted completion time, maximum lateness, and makespan. T. E. Cheng et al. [21] studied a two agents scheduling problem, where the objective was to minimize the total weighted number of tardy jobs for both agents. Lee et al. [22] proposed several approximation algorithms for solving two agents scheduling problems, where the objective was to minimize total weighted completion time. A number of other papers considered the learning effect on processing time [23–25].

The Problem 1 studied in this paper was considered by Alessandro Agnetis et al. [17]. They proved that this problem is NP-hard and proposed a Lagrangian based branch and bound method that can solve the problem up to 60 jobs.

To the best of our knowledge, there is no heuristic or meta-heuristic algorithm proposed for these three problems. Also, we believe that the Problem 2 and Problem 3 considered in this paper have never been considered in the literature. The aim of this paper is to introduce three new problems and develop heuristics and meta-heuristics and to introduce some benchmark problem instances for future research.

## 3. Problem description and notation

This section provides a description of the problems considered in this paper. The jobs from two agents (*agent A* and *agent B*) are

scheduled at a single machine which is responsible for processing a number of jobs that are supplied by two agents. The pre-emption of a job is not allowed. Each job has a given processing time and a weight associated with it. The problem considered in this paper involves finding a sequence for these jobs in such a way that the objective function of agent A is minimized, while keeping the objective function of agent B within a pre-specified level. We use the following terminology to provide mathematical formulation for the problem.

- $n_A/n_B$  number of jobs in agent A/agent B, respectively
- $n$  total number of jobs to be processed,  $n = n_A + n_B$
- $l$  index for A-jobs;
- $m$  index for B-jobs;
- $p$  the position of job in a sequence,  $p = \{1, 2, \dots, n\}$
- $J^A/J^B$  job set of agent A/agent B, respectively, where  $J^A = \{J_1^A, J_2^A, J_3^A, \dots, J_{n_A}^A\}$ ;  $J^B = \{J_1^B, J_2^B, J_3^B, \dots, J_{n_B}^B\}$ . We call “A-job” and “B-job” the jobs from two agents.
- $p_l^A/p_m^B$  processing time of each job from agent A/agent B, respectively.
- $l = 1, \dots, n_A, m = 1, \dots, n_B$
- $w_l^A/w_m^B$  weight of jobs in agent A/agent B, respectively.
- $l = 1, \dots, n_A, m = 1, \dots, n_B$
- $\delta_l^A/\delta_m^B$  density of jobs in agent A/agent B, respectively.
- $\delta_l^A = w_l^A/p_l^A, \delta_m^B = w_m^B/p_m^B$
- $C_l^A/C_m^B$  completion time of jobs in agent A/agent B, respectively.
- $\sigma$  ordered set of jobs already scheduled.
- $f^A(\sigma)$  for given sequence  $\sigma$ , the value of objective function for agent A.
- $f^B(\sigma)$  for given sequence  $\sigma$ , the value of objective function for agent B.
- $Q$  upper bound, a constant number.
- $TCT_A/TCT_B$  total completion time of A-jobs and B-jobs, respectively.
- $TWCT_A/TWCT_B$  total weighted completion time of A-jobs and B-jobs, respectively.

We now consider the following three problems:

Problem 1:

$$1 | \sum_{m=1}^{n_B} w_m^B C_m^B \leq Q | \sum_{l=1}^{n_A} w_l^A C_l^A$$

Problem 1 considers finding a schedule which minimizes the total weighted completion time of the A-jobs such that the maximum total weighted completion time of the B-jobs is within the upper bound  $Q$ . By setting weights of all jobs to one, the problem reduces to  $1 | \sum_{m=1}^{n_B} C_m^B \leq Q | \sum_{l=1}^{n_A} C_l^A$ , which has been proved to be NP-hard by Agnetis et al. [9]. Therefore, Problem 1 is also NP-hard.

We now consider Problem 2 and Problem 3 as follows.

Problem 2:

$$1 | \sum_{m=1}^{n_B} C_m^B \leq Q | \sum_{l=1}^{n_A} w_l^A C_l^A$$

Problem 3:

$$1 | \sum_{m=1}^{n_B} w_m^B C_m^B \leq Q | \sum_{l=1}^{n_A} C_l^A$$

Problem 2 considers finding a schedule which minimizes the total weighted completion time of the A-jobs such that the maximum total completion time of the B-jobs doesn't exceed the upper bound  $Q$ . Problem 3 considers finding a schedule which minimizes the total completion time of the A-jobs such that the maximum

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