Available online at www.sciencedirect.com
ScienceDirect
Comput. Methods Appl. Mech. Engrg. 334 (2018) 40–55
Computer methods in applied mechanics and engineering
www.elsevier.com/locate/cma

# Isogeometric analysis and harmonic stator–rotor coupling for simulating electric machines

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Received 12 September 2017; received in revised form 25 January 2018; accepted 28 January 2018
Available online 5 February 2018

**Abstract**

This work proposes Isogeometric Analysis as an alternative to classical finite elements for simulating electric machines. Through the spline-based Isogeometric discretization it is possible to parametrize the circular arcs exactly, thereby avoiding any geometrical error in the representation of the air gap where a high accuracy is mandatory. To increase the generality of the method, and to allow rotation, the rotor and the stator computational domains are constructed independently as multipatch entities. The two subdomains are then coupled using harmonic basis functions at the interface which gives rise to a saddle-point problem. The properties of Isogeometric Analysis combined with harmonic stator–rotor coupling are presented. The results and performance of the new approach are compared to the ones for a classical finite element method using a permanent magnet synchronous machine as an example.


*Keywords:* Isogeometric analysis; Harmonic stator–rotor coupling; Electric machines; Finite elements

## 1. Introduction

Isogeometric Analysis (IGA) was first introduced in [1,2] and can be understood as a Finite Element Method (FEM) using a discrete function space that generalizes the classical polynomial one. IGA has already been applied in different fields such as, e.g., mechanical engineering [3] and fluid dynamics [4]. A more elaborated overview of relevant application fields can be found in [5]. In this paper, we propose the application of the concepts of IGA to electric machine simulation. According to IGA, the basis functions commonly used in Computer Aided Design (CAD) for geometry construction, i.e. B-Splines and Non-Uniform Rational B-splines (NURBS), are used as the basis for the solution spaces in combination with the classical FEM framework. IGA uses a global mapping from a reference domain to the computational domain and does not introduce a triangulation thereof. As a consequence, it is possible to represent CAD geometries exactly, even on the coarsest level of mesh refinement.

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https://doi.org/10.1016/j.cma.2018.01.047
0045-7825/

The possibility to parametrize circular arcs (and other conic sections) without introducing geometrical errors is of particular interest for electric machine simulation since it guarantees an exact representation of the air gap, independently of the mesh resolution. Furthermore, thanks to the properties of Isogeometric basis functions, IGA solutions have a higher global regularity with respect to their FEM counterparts. The inter-element smoothness of the latter is typically restricted to $C^0$. Moreover, IGA features a better accuracy with respect to the number of degrees of freedom compared to FEM [2,6,7]. Both advantages are of great importance for an accurate simulation of electric machines. For example, torques and forces are often calculated by the Maxwell's stress tensor evaluated in the air gap, in which case the obtained results are very sensitive to the representation and discretization of the air gap [8]. This paper also tackles the problem of stator–rotor coupling which arises by our choice of IGA.

The application of IGA for electric machine simulation is illustrated using a 2D magnetostatic formulation including the treatment of an angular displacement between stator and rotor. A further extension of the formulation to non-linear models [9], to time-harmonic [10] and transient formulations [11] and to the 3D case is straightforward.

The structure of the paper is as follows. We first introduce the 2D model commonly used to describe electric machines. We discuss how IGA is used to discretize the model. Section 3 first presents a naive domain decomposition approach for stator–rotor coupling and then develops a Mortar coupling strategy based on harmonic functions that is in focus of this work. A simplified example is used to analyze the convergence and stability properties of the harmonic stator–rotor coupling. Finally, we apply the proposed method for simulating a permanent magnet synchronous machine (PMSM). The results are compared to a lowest order FEM.

## 2. IGA electric machine model

Electromagnetic fields are described by Maxwell's equations. For electric machines, valuable results can already be obtained using a magnetostatic formulation, i.e., a subset of Maxwell's equations where the eddy currents and displacement currents are neglected [12,13]. The discretization of the resulting set of partial differential equations by FEM requires the use of Nédélec elements where the degrees of freedom are allocated on the edges of the mesh [14]. Proper B-Spline approximation spaces as a counterpart to Nédélec elements in an IGA context were introduced by Buffa et al. [15]. However, for electric machine simulation, it is often sufficient to model a 2D cross section of the geometry. Under these assumptions, Maxwell's equations reduce and combine into a Poisson equation on the computational domain $\overline{\Omega} = \overline{\Omega}_{\mathrm{rt}} \bigcup \overline{\Omega}_{\mathrm{st}}$ (Fig. 1)

$$-\nabla \cdot (\nu \nabla A_z) = \underbrace{J_{\mathrm{src}} + J_{\mathrm{pm}}}_{J_z}, \tag{1a}$$

where $\nu = \nu(x, y)$ is the reluctivity (the inverse of the permeability), assumed to be linear and isotropic, and $A_z = A_z(x, y)$ is the $z$-component of the magnetic vector potential $\vec{A}$. The current densities exciting the coils of the machine and the magnetization current densities related to the permanent magnet (PM) in the rotor are depicted by $J_{\mathrm{src}} = J_{\mathrm{src}}(x, y)$ and $J_{\mathrm{pm}} = J_{\mathrm{pm}}(x, y)$, respectively. Eq. (1) is accompanied by Dirichlet boundary conditions at the outer stator and inner rotor boundary $\Gamma_{\mathrm{d}}$ (see Fig. 1) and anti-periodic boundary conditions at the boundary parts $\Gamma_{\mathrm{l}}$ and $\Gamma_{\mathrm{r}}$ (see Fig. 1), i.e.,

$$A_z|_{\Gamma_{\mathrm{d}}} = 0, \tag{1b}$$

$$A_z|_{\Gamma_{\mathrm{l}}} = -A_z|_{\Gamma_{\mathrm{r}}}. \tag{1c}$$

Note that the air gap itself is split into two parts by $\Gamma_{\mathrm{ag}}$. The inner part of the air gap is included in $\overline{\Omega}_{\mathrm{rt}}$ and the outer part is included in $\overline{\Omega}_{\mathrm{st}}$.

The solution field is discretized by a linear combination of scalar basis functions $w_j(x, y)$, i.e.,

$$A_z(x, y) \approx \sum_{j=1}^{N_{\mathrm{DoF}}} u_j w_j(x, y), \tag{2}$$

where

$$\mathbf{u}^\top = [u_1, \ldots, u_{N_{\mathrm{DoF}}}],$$

is the vector of degrees of freedom. Applying the Galerkin approach results in the system of equations

$$\mathbf{K}\mathbf{u} = \mathbf{j}, \tag{3}$$