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## Pressure and fluid-driven fracture propagation in porous media using an adaptive finite element phase field model

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#### Abstract

This work presents phase field fracture modeling in heterogeneous porous media. We develop robust and efficient numerical algorithms for pressure-driven and fluid-driven settings in which the focus relies on mesh adaptivity in order to save computational cost for large-scale 3D applications. In the fluid-driven framework, we solve for three unknowns pressure, displacements and phase field that are treated with a fixed-stress iteration in which the pressure and the displacement–phase-field system are decoupled. The latter subsystem is solved with a combined Newton approach employing a primal–dual active set method in order to account for crack irreversibility. Numerical examples for pressurized fractures and fluid filled fracture propagation in heterogeneous porous media demonstrate our developments. In particular, mesh refinement allows us to perform systematic studies with respect to the spatial discretization parameter.

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### 1. Introduction

Crack propagation in brittle and porous media is currently one of the major research topics in mechanical, energy, and environmental engineering. In this paper, we concentrate specifically on fracture propagation in three dimensional heterogeneous porous media. We consider a variational approach for brittle fracture introduced by Francfort and Marigo [1] that is formulated in terms of a thermodynamically-consistent phase field technique; see Miehe et al. [2]. Other approaches for treating pressurized fracture include the following: cohesive zone finite elements (CZ-FEM) [3], displacement discontinuity methods (DDM) [4–6], partition-of-unity methods and closely related XFEM/GFEM (extended and generalized finite elements) methods [7–13]. Boundary element methods have been employed in

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[14,15], and peridynamics for hydraulic fracturing has been considered in [16]. Discrete networks of fluid filled fractures have been investigated in [17–21].

Our motivations for employing a phase field model are that fracture nucleation, propagation, kinking, and curvilinear paths are automatically included in the model; post-processing of stress intensity factors and remeshing resolving the crack path are avoided. Furthermore, the underlying equations are based on continuum mechanics principles that can be treated with adaptive Galerkin finite elements. In fact, variational and phase field formulations for fracture are active research areas as attested in recent years; see Bourdin et al. [22,23], Miehe et al. [24,2,25], Borden et al. [26], Artina et al. [27], Burke et al. [28], Allaire et al. [29], Schlüter et al. [30], Ambati et al. [31] and Mikelić et al. [32,33]. Here, discontinuities in the displacement field across the lower-dimensional crack surface are approximated by an auxiliary phase field function. The latter can be viewed as an indicator function, which introduces a diffusive transition zone between the broken and the unbroken material.

For pressurized fractures in porous media, the pressure is a fixed, given quantity or assumed to be computed [33, 34]. The essential aspects of a phase field-based pressurized-fracture propagation formulation are techniques that must include resolution of the length-scale parameter  $\varepsilon$ , the numerical solution of the forward problem and enforcement of the irreversibility of crack growth. The sum of these requirements leads to a variational inequality. For numerical simulations, a robust computational framework in terms of a quasi-monolithic formulation has been proposed in [35] in which a primal–dual active set method (i.e., a semi-smooth Newton method [36]) is coupled with the Newton solver for the nonlinear forward problem.

Our main attention in this paper is on three-dimensional applications that are challenging because of computational cost. This is especially the case for phase field problems because the resolution of the crack requires (very) fine meshes. Here, uniform refinement is infeasible and we adopt a method proposed in [35] for two-dimensional problems and extend these ideas to three-dimensional applications. The efficiency is shown in terms of pressurized and fluid-filled phase field fractures for which systematic 3D studies including mesh refinements are not present in the literature.

In summary, the goal and novelty of the present paper are systematic studies of computational stability using predictor–corrector mesh adaptivity for three-dimensional pressure and fluid-driven phase field fracture problems. Such studies are essential for better understanding between model and discretization parameters in phase field modeling for the previously mentioned applications. We emphasize that the fluid-filled fracture framework in porous media (with Biot's coefficient  $\alpha = 1$ ) is itself novel where we formulate a fixed-stress iteration for the pressure system coupled to the fully-coupled displacement–phase-field system. Here, the latter system is treated with a primal–dual active set method. This idea is in contrast to the fluid-filled phase field fracture framework presented in [37] in which all equations have been decoupled.

The outline of this paper is as follows: We first state the governing equations in Section 2. Then, we present our main algorithm and adaptive discretization in Section 3. In Section 4, we provide numerical examples that demonstrate the potential of this approach for treating practical engineering applications.

### 2. Mathematical models for pressurized and fluid filled fractures

Let  $\Lambda \in \mathbb{R}^d$ , d = 2, 3 be a smooth open and bounded computational domain with Lipschitz boundary  $\partial \Lambda$  and let [0, T] be the computational time interval, T > 0. We assume that the crack C is contained compactly in  $\Lambda$ . Here, we emphasize that the crack is seen as a thin three-dimensional volume where the thickness is much larger than the pore size of the porous medium. The displacement of the solid and diffusive flow in the porous medium are modeled in  $\Omega = \Lambda \setminus \overline{C}$  by the classical quasi-static elliptic-parabolic Biot system for a linear elastic, homogeneous, isotropic, porous solid saturated with a slightly compressible viscous fluid for every  $t \in (0, T]$ .

First, we start from the constitutive equation for the Cauchy stress tensor  $\sigma^{por}$ ,

$$\sigma^{por}(\mathbf{u}, p) - \sigma_0 = \sigma(\mathbf{u}) - \alpha(p - p_0)I \quad \text{in } \Omega \times (0, T],$$
(1)

where  $\mathbf{u} : \Omega \times [0, T] \to \mathbb{R}^d$  is the solid's displacement,  $p : \Omega \times [0, T] \to \mathbb{R}$  is the fluid pressure,  $\alpha \in [0, 1]$  is the Biot coefficient, *I* is the identity tensor,  $\sigma_0$  and  $p_0$  are the given initial values when t = 0, which are set to be zero for simplicity in this paper. The effective linear elastic stress tensor  $\sigma := \sigma(\mathbf{u})$  is

$$\sigma(\mathbf{u}) = \lambda(\nabla \cdot \mathbf{u})I + 2Ge(\mathbf{u}),\tag{2}$$

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