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# Exponentially accurate nonconforming least-squares spectral element method for elliptic problems on unbounded domain

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**Abstract**

Non-conforming approximation methods are becoming increasingly popular because of the potential to apply to multi-material and multi-model analysis for both bounded and unbounded domains. In this paper, we present a least-square approximation based method to solve the one or two dimensional elliptic problems on an unbounded domain. The method gives exponential accuracy and shows superior performance when compared to other numerical methods. Differentiability estimates and the main stability estimate theorem, using a non-conforming spectral element method, are also discussed. The exponential convergence rate of the proposed method is also shown through rigorous error estimate and specific numerical examples.

*Keywords:*
Least-squares method, nonconforming, spectral element method, unbounded domain, preconditioner, parallel computers, exponential accuracy

## 1. Introduction

The approach of artificial boundary conditions, on a fictitious finite external boundary, is widely used to solve differential equations on unbounded domains. In general, there are many issues related to the setting of artificial boundary conditions, used in many areas of scientific computing, such as acoustics, electrodynamics, solid mechanics, and fluid dynamics. The external problems in computational fluid dynamics have a wide class of formulations. For example, the overall quality and performance of numerical algorithms and interpretation of the results in computational fluid dynamics depend critically on the proper treatment of external boundaries.

In the last three decades, spectral methods have been widely used for solving partial differential equations (PDEs) arising in engineering and science. Bilinova [5] proposed spectral methods. The first implementation of spectral methods was given by Silberman [47], but was abandoned later. Orszag [37] and Eliasen et al. [17] resurrected it again. Gottlieb et al. [20] provided the first unified mathematical formulation of the theory of modern spectral methods for the numerical solution of partial differential equations. Multi-dimensional discretizations were formulated as tensor products of one-dimensional constructs in separable domains. Since then, spectral methods were extended to a broader class of problems. The book of Canuto et al. [9] focuses on fluid dynamics algorithms and includes both practical as well as theoretical aspects of global spectral methods. A companion book by Canuto et al. [10] is focused on the essential aspects of spectral methods on separable domains. The book by Karniadakis et al. [19], deals with many important practical aspects of computations using spectral methods and summarize the recent research in the subject. The other two useful books by Shen and coworkers [44, 45] are focused on many important practical aspects of computations using spectral methods and summarize the recent research in the subject of unbounded domains.

The use of spectral methods, in the solution of PDEs on unbounded domains, can be divided into four major sections. First, through truncation of the unbounded domain to bounded domain and solve the partial differential equation using artificial or transparent boundary [18, 21]. The second approach for unbounded domain is based on approximation by classical orthogonal system, for example, Laguerre or

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