Available online at www.sciencedirect.com
ScienceDirect
Comput. Methods Appl. Mech. Engrg. 305 (2016) 910–935
Computer methods in applied mechanics and engineering
www.elsevier.com/locate/cma

Interval and random analysis for structure–acoustic systems with large uncertain-but-bounded parameters

Shengwen Yin, Dejie Yu*, Hui Yin, Baizhan Xia

State Key Laboratory of Advanced Design and Manufacturing for Vehicle Body, Hunan University, Changsha, Hunan, 410082, People's Republic of China

Received 22 October 2015; received in revised form 17 February 2016; accepted 21 March 2016
Available online 30 March 2016

Abstract

For the response analysis of the structure–acoustic system with uncertain-but-bounded parameters, three bounded uncertain models are introduced. One is the bounded random model in which all of the uncertain-but-bounded parameters are described as bounded random variables with well defined probability distribution. The second one is the interval model in which all of the uncertain-but-bounded parameters are described as interval variables due to the limited information. The third one is the bounded hybrid uncertain model in which the interval variables and the bounded random variables exist simultaneously. Based on the parametric Gegenbauer polynomial, which is formulated for bounded random model recently, the *Gegenbauer Series Expansion Method* (GSEM) is developed for the response prediction of the structure–acoustic system under these three bounded uncertain models. Within GSEM, the response of these three bounded uncertain models of the structure–acoustic system can be approximated by the unified Gegenbauer Series with different values of polynomial parameter. Then, the interval and random analysis for these three bounded uncertain models of the structure–acoustic system are conducted on the basis of Gegenbauer series. Owing to the orthogonal property of Gegenbauer polynomial, the analytical solution of the expectation and variance of Gegenbauer series with respect to the bounded random variables can be readily obtained. The bounds of Gegenbauer series with respect to the interval variables are determined by the Monte Carlo simulation. The GSEM is applied to solve a shell structure–acoustic system under these three bounded uncertain models. Inspired by the convergence behavior of GSEM, the relative improvement criterion is established to estimate the required retained order of Gegenbauer Series for large uncertain problems. The results on numerical examples show that GSEM with the estimated retained order can achieve a prescribed accuracy and good efficiency for structure–acoustic systems with large uncertain-but-bounded parameters.

Keywords: Gegenbauer series expansion; Relative improvement criterion; Interval model; Bounded random model; Bounded hybrid uncertain model; Structure–acoustic system

* Corresponding author. Tel.: +86 731 88821915; fax: +86 731 88823946.
E-mail address: djyu@hnu.edu.cn (D. Yu).

http://dx.doi.org/10.1016/j.cma.2016.03.034

1. Introduction

Vibro-acoustic analysis of structure–acoustic systems has undergone a rapid development in engineering, especially in the field of the automobile, train, airplane and other transports, where the coupled interaction of structure and acoustic field is playing a more and more significant role. Traditional numerical methods for the response analysis of the structure–acoustic system are based on deterministic parameters [1]. However, due to aggressive environment factors, inevitable manufacturing errors and incomplete knowledge, uncertainties associated with material properties, applied loads and other parameters are always involved in structure–acoustic systems. Without considering these uncertainties, the vibro-acoustic analysis for structure–acoustic systems may be unreliable. Up to now, the random model is still considered as the most valuable mathematical model to treat with the uncertainties existing in engineering practices. In the random model, the uncertain parameter is treated as random variable whose *probability density function* (PDF) is well defined in its variational range [2]. For the response analysis of random systems, a lot of random methods have been developed. Mainly, there are three kinds of numerical methods for dealing with random models, that is the *Monte Carlo Method* (MCM) [3,4], the random perturbation method [5,6], and the random orthogonal polynomial approximation method [7–12]. Among these three methods, MCM can achieve a best accuracy and is relatively simple in formulations. However, because of the tremendous computational effort, MCM is often used as the reference approach for validating the accuracy of other probability approaches. The random perturbation method is the most efficient method for random problems, but it may only be applicable for systems with small uncertainty level. Based on the orthogonal polynomial theory, some random orthogonal polynomial approximation methods are proposed for random problems with large uncertainties. The random orthogonal polynomial approximation method is free from small perturbation assumption and the efficiency is much higher than MCM.

Originally, the orthogonal polynomial approximation method was formulated for standard Gaussian distribution with Hermite polynomials [8]. The generalized *Polynomial Chaos* (PC) is then developed to model the random parameter which belongs to several typical probability distributions [10]. Recently, there is an increasing interest in representing the more general non-Gaussian distributions. Shields used a translate process to describe the non-Gaussian distribution based on the standard Gaussian distribution and the associated Hermite polynomial [11]. The drawback of this translate process is that their translated distributions are infinite. For uncertain problems with bounded random variables whose variational ranges and PDFs are well defined, a random Gegenbauer series expansion method is proposed by Ma et al. [12,13]. In random Gegenbauer series expansion method, a λ-PDF model and its derivative model are established based on the Gegenbauer polynomials. The derivative λ-PDF model has the flexibility to fit the bounded random variables with any mono-peak or mono-valley PDFs. Thus, the Gegenbauer series expansion method has a good prospect in practical engineering. Note that some polynomials of generalized PC expansion can also handle the bounded random problems. But so far, the application of generalized PC expansion mainly focuses on some typical bounded random problems, such as the application of Legendre polynomial for continuous uniform distribution and the application of Jacobi polynomial for Beta distribution [14,15]. It can be seen that the probabilistic approaches mentioned above have achieved a great success in the field of uncertain analysis. However, the probabilistic approaches give reliable results only when sufficient statistical data are available to construct the PDF of uncertain parameters.

To overcome the shortcoming of the probabilistic approach, the non-probabilistic approaches have emerged as the alternative tools for the uncertain problems with limited information [16,17]. Interval approach is one of the typical non-probabilistic approaches. In the interval model, the uncertain parameter is treated as interval variable and only the bounds of the uncertain parameter are required to represent the uncertain property. For interval analysis, the MCM [18], the perturbation theory [19–21] and orthogonal polynomial theory [22–24] can be also employed. It has been seen in literature [22] that the Chebyshev interval method has a better accuracy than the interval perturbation method, and the efficiency of the Chebyshev interval method is much higher compared with MCM. In addition to the above interval methods, some interval methods such as the interval factor method [25,26] and the vertex method [27] etc., have been proposed to solve the interval problems in some special cases.

The probability approach and the interval approach are appropriate for the uncertain problems with random variables and interval variables respectively. However, the random variables and interval variables may exist simultaneously in some cases. For the analysis of uncertain engineering problems with both random and interval parameters, a hybrid uncertain method was proposed by Elishakoff and Colombi [28]. The hybrid uncertain model has been widely