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Computational vademecums for the real-time simulation of haptic collision between nonlinear solids[☆]

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Abstract

In this paper a novel strategy is presented for the real-time simulation of contact between non-linear deformable solids at haptic feedback rates. The proposed method is somehow related to the *Voxmap Pointshell* method for two deformable solids. Its novelty and crucial advantages over existing implementations of this algorithm come from the intensive use of *computational vademecums*. These are in essence a pre-computed solution of a parametric model in which every possible situation during the on-line phase of the method has been considered through the introduction of the appropriate parameters. Such a high-dimensional parametric model is efficiently solved by using Proper Generalized Decompositions (PGD) and stored in memory as a set of vectors. The paper presents a thorough description of the developed algorithm together with some examples of its performance.

Keywords: Real time; Contact; Model reduction; Proper Generalized Decomposition; Parametric models; Computational vademecums

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1. Introduction

Computational contact mechanics [1] constitutes nowadays a very active field of research due to its inherent difficulty, associated to the highly non-linear character of its models. But when we deal with non-linear solids and, in addition, real-time response is required, the problem becomes extremely burdensome and has generated a plethora of

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publications searching for a good compromise between accuracy and time to response, see for instance [2–5], to name but a few of the available references.

Haptic peripherals have become very popular for *augmented* simulation in immersive environments, particularly for education purposes and games. They have been used notably in medical environments as an essential tool for the education in minimally invasive procedures [6–13] and for training of complex industrial processes. In particular, aircraft and automobile industries have recently incorporated virtual reality to the design process to see whether a mechanical part could be located at a particular position in the plane in order to facilitate manufacturing and maintenance processes [14–16].

The main difficulty associated to haptic peripherals (those with force feedback) is that they need, to provide with a realistic sense of touch, a feedback response in the order of 500 Hz to 1 kHz. What this means in practice is that, very much like some 25 frames per second are needed in cinemas to provide the spectator with a continuous sensation of movement, in haptic environments nearly one thousand simulations per second must be carried out to provide the user with a continuous sense of touch. 1 kHz is roughly the free-hand gesture frequency [17].

The reader will readily understand the difficulties associated with the simulation of non-linear solids (soft living tissues are frequently assumed to be hyperelastic, possibly with fiber reinforcement [18,19]) under such astringent requirements. If, in addition, the possibility of contact between deformable solids is taken into account, the 1 kHz constraint is even more difficult to fulfill.

Contact mechanics simulations under real-time restrictions have been tackled from a variety of approaches. One of the earliest and most popular is that of constructing Bounding Volume Hierarchies (BVH) [5], consisting in associating each node in a tree with a subset of the primitives (polygons, NURBS, etc.) defining the boundary of the object. Other approaches for real time include the use of stochastic contact detection [20], based on the assumption that the perceived realism of contact detection depends more of the real-time response than in the accuracy itself of the simulation.

One of the most popular families of real-time contact detection algorithms is based on the use of distance fields [21,4]. Distance fields (level sets) constitute a very convenient way of representing very intricate geometries for contact detection, but have been traditionally considered as non-apt for real-time contact simulation between deformable solids, since the distance field must be updated according to the deformation of one of the solids [5].

If we restrict ourselves to the problem of real-time simulation of hyperelastic solids, several approaches have been accomplished in the literature. Besides the obvious choice of considering a purely elastic material, but which renders very poor results in terms of visual realism in the presence of large strains, the first approaches considered multi-resolution methods [22,23]. Very popular in the last years are the methods based on the use of explicit finite element implementations [24], possibly based on parallelization by using general-purpose Graphics Processing Units (GPUs) [25,6,26]. Due to the inherent complexity of the objective, techniques based upon model order reduction methods have also recently reached some popularity. Among them, we can cite [27–32]. In general, all these last references are based on the use of Proper Orthogonal Decomposition (POD) techniques [33–35], known also as Principal Component Analysis (PCA). These techniques employ a statistical treatment of the results obtained for complete, *similar* problems to the one at hand to construct a set of global, Ritz-like, basis functions that are optimal, with respect to some measure, for the already solved, complete problems. They are then used for the problem under consideration in the hope that, if the modifications with respect to the original problems are small, they will be also a good choice for it.

This approach presents some drawbacks. For instance, the choice of *similar*, complete problems, after which to obtain the set of global basis, is not an easy task [36]. In addition, model reduction methods loose many of their advantages if we deal with non-linear problems, since they need for the reconstruction of the full tangent stiffness matrix in order to obtain a consistent linearization of the problem. Although several approaches have been developed to deal with this difficulty, no definitive response has been found [28,37,38].

As an alternative to POD techniques, Proper Generalized Decomposition (PGD) techniques can be seen as a sort of *a priori* model order reduction method, in the sense that no complete problem must be solved in order to obtain the set of global basis functions. The origin of PGD dates back to the so-called *radial loading* approximation of the LArge Time INcrements (LATIN) method [39]. It consisted, essentially, of a space–time separated representation of the solution that very much resembles that of POD, but obtained completely in an *a priori* fashion.

Some years later, F. Chinesta employed a similar separated representation for the solution of high-dimensional problems arising from the kinetic theory of complex, non-Newtonian flows [40,41]. Once both approaches have been identified as belonging to one single method for the model order reduction of PDEs, the field of application of PGD