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Equilibrium reconstruction in an iron core tokamak using a deterministic magnetisation model ☆

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ABSTRACT

In many tokamaks ferromagnetic material, usually referred to as an iron-core, is present in order to improve the magnetic coupling between the solenoid and the plasma. The presence of the iron core in proximity to the plasma changes the magnetic topology with consequent effects on the magnetic field structure and the plasma boundary. This paper considers the problem of obtaining the free-boundary plasma equilibrium solution in the presence of ferromagnetic material based on measured constraints. The current approach employs a model described by O'Brien et al. (1992) in which the magnetisation currents at the iron–air boundary are represented by a set of free parameters and appropriate boundary conditions are enforced via a set of quasi-measurements on the material boundary. This can lead to the possibility of overfitting the data and hiding underlying issues with the measured signals. Although the model typically achieves good fits to measured magnetic signals there are significant discrepancies in the inferred magnetic topology compared with other plasma diagnostic measurements that are independent of the magnetic field. An alternative approach for equilibrium reconstruction in iron-core tokamaks, termed the deterministic magnetisation model is developed and implemented in EFIT++. The iron is represented by a boundary current with the gradients in the magnetisation dipole state generating macroscopic internal magnetisation currents. A model for the boundary magnetisation currents at the iron–air interface is developed using B-Splines enabling continuity to arbitrary order; internal magnetisation currents are allocated to triangulated regions within the iron, and a method to enable adaptive refinement is implemented. The deterministic model has been validated by comparing it with a synthetic 2-D electromagnetic model of JET. It is established that the maximum field discrepancy is less than 1.5 mT throughout the vacuum region enclosing the plasma. The discrepancies of simulated magnetic probe signals are accurate to within 1% for signals with absolute magnitude greater than 100mT; in all other cases agreement is to within 1mT. The effect of neglecting the internal magnetisation currents increases the maximum discrepancy in the vacuum region to >20mT, resulting in errors of 5%–10% in the simulated probe signals. The fact that the previous model neglects the internal magnetisation currents (and also has additional free parameters when fitting the measured data) makes it unsuitable for analysing data in the absence of plasma current. The discrepancy of the poloidal magnetic flux within the vacuum vessel is to within 0.1Wb. Finally the deterministic model is applied to an equilibrium force-balance solution of a JET discharge using experimental data. It is shown that the discrepancies of the outboard separatrix position, and the outer strike-point position inferred from Thomson Scattering and Infrared camera data are much improved beyond the routine equilibrium reconstruction, whereas the discrepancy of the inner strike-point position is similar.

1. Introduction

Accurate knowledge of the magnetic field structure in a tokamak is a prerequisite first step for the analysis and control of plasma discharges. In order to improve the magnetic coupling between the solenoid and the plasma many tokamaks (e.g. JET [1], ISTTOK [2], Tore Supra [3], STOR-M [4], TEXT [5]) incorporate a ferromagnetic core, usually referred to as an iron core. The

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presence of the iron core in proximity to the plasma changes the magnetic topology with consequent effects on the plasma boundary shape. From the numerical point of view the presence of the iron core represents a complication by adding an additional non-linearity to the calculation of equilibrium force-balance. This calculation in tokamaks, based on measured constraints, known as *equilibrium reconstruction* is carried out by codes such as EFIT [6,7], EQUAL [8,9], CLISTE [10] and EQUINOX [11–13].

This paper considers the case of the JET tokamak using EFIT++[14]. The code is used for equilibrium reconstruction on JET between discharges (i.e. during *intershot* operation) and in subsequent detailed interpretive analyses. The EFIT++ code, based on the algorithm of EFIT [6], is machine-agnostic, written mainly in C++ and was developed at JET and the Culham Centre for Fusion Energy. Originally the EFIT++ code implemented a magnetisation model based on [1]. For the computation of equilibrium reconstructions carried out between discharges the boundary magnetisation currents (at the iron–air interface) are represented by a set of free parameters and appropriate boundary conditions are enforced via a set of quasi-measurements on the material boundary. These quasi-measurements are used in a χ^2 minimisation together with other measurements (typically magnetic flux, local magnetic field and power supply currents) to determine the equilibrium reconstruction based only on magnetic data. This is the so-called EFIT++ fitting model. The advantage of running EFIT++ in this manner is that it is fast, typically 0.1–1 s per time-slice. The rapid execution time is a result of the boundary iron magnetisation being represented by comparatively few discrete piecewise constant currents. This is possible because the iron currents are adjusted during the χ^2 minimisation, the purpose being to get a strong fit to the diagnostics in the proximity of the plasma. Furthermore, only the iron boundary closest to the plasma is included. Including the boundary currents as additional degrees of freedom in the χ^2 fit can significantly increase the number of free parameters, particularly because the number of boundary currents is generally much greater than the number of free parameters from the other current sources. This can lead to the possibility of overfitting the data and hiding underlying issues with the measured signals: although the EFIT++ fitting model typically achieves good fits to measured magnetic signals there are significant discrepancies in the inferred magnetic topology compared with other plasma diagnostic measurements. In this work we present an alternative approach for equilibrium reconstruction in iron core tokamaks, termed the deterministic model. The iron is represented by a boundary current with the gradients in the magnetisation dipole state generating macroscopic internal magnetisation currents. A model for the boundary magnetisation currents at the iron-air interface is developed using B-Splines enabling continuity to arbitrary order; internal magnetisation currents are allocated to triangulated regions within the iron, and a method to enable adaptive refinement is implemented. This model is strongly based on the underlying physics described by Maxwell's equations coupled with the constitutive relations of the ferromagnetic materials. Therefore, the magnetisation currents, both the boundary and internal currents, are enforced exactly and do not appear as quasi-measurements in the χ^2 minimisation. Compared to the fitting model there are no quasi-measurements enabling improved interpretation of the real measurements. A practical implication of this is to enable the possibility of carrying out in-vessel calibration of the magnetic diagnostic system to improve the consistency of the model against the experimental data in iron-core tokamaks. Section 2 summarises the key components of the EFIT++ algorithm followed by a derivation of the current distribution in a ferromagnetic material. Section 3 develops a model for the representation of boundary magnetisation currents at the iron–air interface. Section 4 describes the implementation of the internal magnetisation currents. Section 5 validates the deterministic model by comparing it with a synthetic 2-D electromagnetic model of JET. Finally Section 6 describes the use of the EFIT++ deterministic model on an equilibrium force-balance solution of a JET discharge using only magnetic signals; results are compared with other independent diagnostic measurements.

2. EFIT++ equilibrium force-balance algorithm in presence of ferromagnetic material

The EFIT++ algorithm sets out to provide an equilibrium force balance solution that is consistent with measurements whilst taking into account the presence of ferromagnetic material. The flow diagram of the EFIT++ algorithm, valid for both the fitting and the deterministic iron models, is illustrated in Fig. 1a. The programme enters a loop which iterates towards a converged equilibrium force balance solution by successively invoking the magnetisation model, the linearised Grad–Shafranov solver and a least-squares algorithm to update values of poloidal field circuit currents and coefficients of the plasma-based flux functions defined below. Optionally there may be an inner loop to improve the convergence of the magnetisation model. Below we summarise the algorithm.

The state of axisymmetric equilibrium force balance in a tokamak is encapsulated in the Grad–Shafranov equation:

$$\Delta^* \psi_p = -2\mu_0 R J_\phi. \tag{1}$$

The equation, expressed in right-handed cylindrical coordinates (R, ϕ, Z), is written in terms of poloidal flux $\psi_p = RA_\phi$ where A_ϕ is the toroidal component of the magnetic vector potential [15], and toroidal current density, J_ϕ, which can itself be expressed as a function of two plasma-based flux functions, $p(\psi_p)$ and $f(\psi_p)$ and a non-plasma based component $J_{\rm ext}$:

$$J_\phi = R\frac{\partial p}{\partial \psi_p} + \frac{1}{\mu_0 R} f \frac{\partial f}{\partial \psi_p} + J_{ext}(R, Z). \tag{2}$$

The parameter $J_{\rm ext} = J_{\rm pf} + J_{\rm induced} + J_{\rm iron}$ in which $J_{\rm pf}$ is associated with the set of n_{pfc} poloidal field circuit currents $\{I_i^{pfc}\}$; $J_{\rm induced}$ is associated with n_{ind} independent induced currents $\{I_i^{ind}\}$; and $J_{\rm iron}$ is associated with n_{iron} independent currents $\{I_i^{iron}\}$. The first two terms on the right hand side of (2) represent the plasma current flowing in a closed region bounded by a magnetic separatrix. The flux functions have the following generic form:

$$p'(\psi_p) = \sum_{i=1}^{n_\alpha} \alpha_i c_i(\bar{\psi}_p) \tag{3}$$

$$ff'(\psi_p) = \sum_{i=1}^{n_\beta} \beta_i d_i(\bar{\psi}_p) \tag{4}$$

where $\{c_i(\bar{\psi}_p)\}$ and $\{d_i(\bar{\psi}_p)\}$ are sets of basis functions; a typical choice is to use $c_i = d_i = \bar{\psi}_p^{i-1}$ but EFIT++ also permits other choices for example tension splines [16] enable more realistic current distributions in advanced reconstructions. This is out of scope of this paper. Here,

$$\bar{\psi}_p = (\psi_p - \psi_0)/(\psi_b - \psi_0) \tag{5}$$

is the normalised poloidal flux where ψ_b and ψ_0 are the poloidal flux values at the plasma separatrix and at the magnetic axis [6].

Referring to the EFIT++ flow diagram shown in Fig. 1 the *Linearised Grad–Shafranov solver* solves (1) with J_ϕ constructed using $\bar{\psi}_p$ generated at a previous step [6]. The field solution in the region containing the plasma current is solved using the fast 2-D cyclic reduction finite difference method of Buneman [17]. Having computed a solution for $\psi_p(R, Z)$, the loci of the separatrix