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A unified scaled boundary finite element method for transient two-dimensional vibro-acoustic analysis of plate-like structures

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ABSTRACT

This paper develops a unified and efficient simulation technique based on the scaled boundary finite element method (SBFEM) for transient structural–acoustic problems in two dimensions. The structural component can be an assembly of thin to moderately thick unidirectional plates. Each of them is treated as a plane strain problem and the principle of virtual work involving the inertial contribution is applied to derive the scaled boundary finite element equation. Only the longitudinal dimension is discretized with line elements and the solution through the thickness is expressed analytically as a Padé expansion. A variable transformation process together with certain structural assumptions leads to the static stiffness and mass matrices. The acoustic field can be a semi-infinite region, which is first divided by an artificial boundary into a near field and an exterior domain. The former is analyzed using the improved continued-fraction approach and the latter is modeled by the improved doubly-asymptotic open boundary. These formulations are based on velocity potential and established consistently in the SBFEM framework. High-order spectral elements are employed for both the structural and acoustic domains, to which independent discretization schemes are applied. Numerical examples are presented to demonstrate the excellent performances of the proposed technique.

1. Introduction

Plates and shells, widely used in aerospace, marine and automotive industries, are frequently excited by periodic or transient forces when the corresponding mechanical equipment is operating. As a consequence, these structures begin vibrating and radiating sound into the surrounding bounded or unbounded fluid fields, which will not only disturb the crew and passengers, but also possibly cause damage to the related devices and even make a submarine in war exposed to danger. The alleviation or elimination of such issues relies mainly on a scientific understanding of the coupled vibro-acoustic systems in the frequency or time domain, which has stimulated a lot of research work in the context of numerical simulations.

The finite element method (FEM) [1] is most commonly employed due to its efficiency and versatility to model the elastic structure in the analysis of an exterior structural-acoustic problem. For the solution of the unbounded fluid domain, the boundary element method (BEM) [2] is preferred since the Sommerfeld radiation condition is fulfilled exactly by the fundamental solution and only the boundary of the acoustic field, i.e., the outer surface of the structure, needs to be discretized. Therefore, the FEM and BEM are usually combined to analyze the fluid-structure interaction problems. Earlier representative studies on this coupling scheme can be found in [3,4] where the non-uniqueness difficulties arising from the drawback of the classical boundary integral formulation for the exterior domain were handled via the combined Helmholtz integral equation formulation (CHIEF) [5]. Unfortunately, the CHIEF method is satisfactory only at relatively low frequencies conditionally. In contrast, the Burton-Miller formulation [6] is valid for the entire frequency range and has been mostly applied to circumvent the non-uniqueness problem. In fact, these earlier studies [3,4] employed collocation method for the boundary element implementation, leading to nonsymmetric fluid matrices and thus expensive computational cost. In order to derive symmetric matrix systems, several boundary element formulations based on other schemes such as variational principle [7–9], Galerkin method [10] or acoustic reciprocal principle [11] were then presented for the infinite fluid domains of various vibro-acoustic problems. Besides, since the conventional BEM leads to dense matrices, the resulting system matrices from the coupled finite element and boundary element approach are similarly fully populated, which restricts its application to only simple structural-acoustic

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problems. Therefore, fast multipole method [12], adaptive cross approximation approach [13] and hierarchical matrix technique [14], each of which can significantly reduce the memory consumption and computation time when dealing with the matrix-vector products, have been separately introduced in the coupling scheme for efficient vibro-acoustic analyses [15–20]. In spite of so many efforts devoted to improving the efficiency of the coupled finite element and boundary element method, however, it is still computationally expensive for complex structural-acoustic problems and long-time responses due to the nonlocal nature of the BEM both in space and in time.

In contrast to the aforementioned hybrid technique, the approximation model of an exterior fluid-structure interaction problem can be alternatively constructed in a unified finite element framework. Specifically, the unbounded acoustic field is divided by an artificial boundary into a finite computational domain and an infinite residual domain where the former is suitable for a direct finite element analysis and the latter can be simulated by various specific methods. Infinite elements [21] use complex-valued basis functions with outwardly propagating wave-like behavior to represent the exterior complement. The unconjugated infinite element proposed by Burnett [22] was demonstrated to be much more efficient than the BEM in dealing with large structural-acoustic problems while the conjugated infinite element by Astley [23] was shown to be able to perform transient acoustic analysis effectively for a full range of excitation frequencies. However, it has been confirmed that [24] the unconjugated Burnett formulation is valid only for the near field analysis and that the conjugated Astley-Leis scheme performs better in the far field. In addition, the infinite element schemes tend to become ill-conditioned for sufficiently high radial orders unless proper shape functions are selected [25]. Recent applications of infinite elements in conjunction with conventional finite elements to steady and transient analyses of exterior structural-acoustic problems can, respectively, be found in [26,27]. Specifying an absorbing boundary condition on the artificial boundary previously mentioned provides an alternative way to complete the statement of the problem in the computational domain. The Dirichlet-to-Neumann (DtN) map [28,29] is an exact non-reflecting boundary condition whose accuracy can be easily increased theoretically to a desired level with good stability. However, the original DtN boundary condition is global both spatially and temporally, making its finite element implementation unfeasible for large-scale computations over long time intervals. To this end, two sequences of local time-dependent non-reflecting boundary conditions based on the exact DtN version in the frequency domain were developed by Thompson and Pinsky [30] and were incorporated in a time-space finite element formulation for analyzing exterior structural acoustics problems. Also, the DtN FEM is limited to artificial boundaries with simple shapes. Fortunately, the local non-reflecting boundary condition proposed by Bayliss and Turkel [31] can naturally remedy the weaknesses of the DtN boundary condition. Particularly, the Bayliss-Turkel second-order conditions have been incorporated into symmetric or nonsymmetric finite element formulations for solving transient structural-acoustic problems [32–34]. Nevertheless, the Bayliss-Turkel sequence of boundary conditions cannot be implemented up to an arbitrary high-order due to the presence of high-order derivatives and practical high-order local absorbing boundary conditions have been devised [35]. However, these high-order boundaries are only singly asymptotic at the high-frequency limit, which means they may fail in predicting late-time (low-frequency) responses. By contrast, doubly asymptotic approximation (DAA) boundaries [36] exhibit correct asymptotic behaviors at both the low and high frequency limits and provide a smooth transition in the intermediate frequency range. The finite element DAA scheme was originally proposed for fluid-structure interaction problems and has been widely applied in this area since then [37–40]. Whereas, this formulation is spatially nonlocal and the highest order reported is three [40,41].

A common point shared by the aforementioned numerical techniques for exterior vibro-acoustic problems is that the structural components were all modeled by the FEM. Actually, different situations also exist in the literature. A unified BEM was presented by Chen and Liu [42] for the vibro-acoustic interaction analysis of shell-like structures. Compared with the coupled finite element and boundary element method mentioned earlier, this unified formulation possesses advantages in modeling structures with complicated features and in exhibiting the coupling effect from the conforming meshes on the fluid-structure interface. Nevertheless, its computational efficiency suffers inevitably from the fully populated matrices. This issue can be mitigated via the fast multipole method [12], as shown in the subsequent work by Wilkes and Duncan [43]. However, computational burden is still a great concern for this improved BEM. In addition, an efficient scheme for acoustic-structure interaction problems has recently been presented by Cui et al. [44]. In this formulation, the shell structure was simulated by the edge-based smoothed finite element method [45] to achieve a close-to-exact stiffness while the unbounded acoustic field was truncated by an artificial boundary, where the finite computational domain was modeled by the gradient weighted finite element method [46] to reduce the dispersion error and the exterior residual domain was represented by the exact DtN boundary condition [28]. As stated earlier, the non-locality of the DtN map will greatly restrict the computational efficiency of the coupled technique in analyzing complex vibro-acoustic problems and in calculating long-time responses.

Above is a literature review on the numerical methods for coupled vibro-acoustic analyses of plate-like and shell-like structures submerged in unbounded fluids, with the emphasis laid on commenting the simulation schemes for the infinite acoustic fluids. However, it should also be noted that most of the numerical formulations for the structural components were based on the classical plate or shell theories, thus limiting their applications to only thin structures. Otherwise, the utilization of the first-order shear deformation theory will restrict the procedures to relatively thick structures due to the emergence of shear locking [47]. In order to make these approaches applicable to both thin and moderately thick structures, the locking problems have to be addressed by additional techniques such as reduced integration [47] and discrete shear gap method [48], which were respectively adopted in the aforementioned studies by Pinsky et al. [33] and Cui et al. [44]. Moreover, the majority of the above papers associated with structural-acoustic analyses applied a conforming mesh on the fluid-structure interface, which renders the constructed numerical techniques to be less flexible for these problems. Additionally, some of the mentioned formulations paid special attention to steady-state analyses and the transformation of them into time domain for efficient transient analyses may be impracticable. All of these observations and investigations indicate that each of the existing numerical techniques for exterior structural-acoustic problems has its merits and drawbacks and, more importantly, that novel approaches with prominent advantages in various aspects including accuracy, applicability, computational efficiency and numerical robustness are greatly required.

The scaled boundary finite element method (SBFEM) [49] is a semi-analytical procedure initially proposed for the dynamic analysis of unbounded domains. Only the boundary of the problem domain needs to be discretized and the radiation condition at infinity is satisfied exactly, while no fundamental solution is required unlike the BEM. Compared with the FEM, solutions of the SBFEM are analytical in the radial direction. The SBFEM was successfully applied during its early stage to soil-structure