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Reverse engineering of deep drawn components with an isogeometric framework

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ABSTRACT

In this work, the framework of isogeometric shell analysis is applied to the simulation of the dynamic behaviour of thin components measured by a 3D laser scanner. To this end, a B-spline surface is reconstructed to describe the mid-surface of the components and is used for the analysis. The exact thickness distribution is also described through the B-spline basis functions and is considered in the numerical integration. Two components manufactured through deep drawing are studied and the surface fitting procedure is adapted to their particular geometry. The results are finally validated through experimental testing of these components.

1. Introduction

Isogeometric analysis (IGA) is a numerical modelling methodology that has been recently introduced to bridge the gap between computer aided design (CAD) and numerical analysis [1,2]. As the word isogeometric suggests, the same basis functions that describe the geometry of a component are used directly for the analysis, avoiding the meshing procedure that is needed in the classical finite element approach.

In the original version of IGA these functions are non-uniform rational B-splines (NURBS) [3], although more recently other methodologies have also been considered in order to improve the geometric flexibility of IGA, including T-splines [4–6], locally refined B-splines [7], hierarchical [8] and truncated hierarchical [9] B-splines, subdivision schemes [10,11] and other spline methodologies based on unstructured meshes [12–14], to name some of the most relevant contributions.

Besides avoiding the meshing procedure, IGA allows one to easily increase the order of the basis functions and, therefore, it is particularly well suited for problems governed by high-order partial differential equations [15–19]. In particular, in the field of structural mechanics, isogeometric shell formulations have been recently developed following both the Reissner-Mindlin [20] and the Kirchhoff-Love theory [21]. Since then, other formulations and applications emerged, see e.g. [22–27].

The Reissner-Mindlin formulation is normally employed for the description of thick shells where, as a rule of thumb, the ratio between the curvature radius of the surface R and the thickness t is smaller than 20 and transverse shear deformations need to be taken into account. On the other hand, for thin shells ($R/t \geqslant 20$), the Kirchhoff-Love formulation can be used, which assumes that transverse shear deformations are negligible.

In practice, although many shell structures could be regarded as thin shells, Reissner-Mindlin models are a lot more common in the FE community. In fact, these models assume also rotations as degrees of freedom (DOFs) and can be implemented using the classical FE C^0 inter-element continuity. This is not the case for Kirchhoff-Love models because they do not consider rotational DOFs and, therefore, need C^1 inter-element continuity. This continuity is not easily obtained when employing conventional Lagrangian shape functions, but needs more complex and expensive implementations, see for instance [28].

Kirchhoff-Love shell formulations became more popular with the introduction of IGA, because NURBS functions are in general much smoother and allow by construction a higher inter-element continuity. On removing rotational DOFs, the size of the resulting system matrix is significantly reduced and its units are consistent, which improves the condition of the mass and stiffness matrices. In addition, the shear locking issues typically encountered in (low-order) Reissner-Mindlin elements [29] are also avoided. For these reasons, the Kirchhoff-Love formulation is considered in this work while the interested reader is referred to [30] for a more detailed overview on both formulations. Another important advantage

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coming from IGA with respect to the classical FE approach, is that the geometrically exact character of IGA can be of crucial advantage for shell models, which can be very sensitive to small imperfections [31]. However, for NURBS based IGA, the aforementioned considerations on the C^1 continuity are valid for single patch geometries, while for complex models constituted by non-conforming multi-patch configurations, the C^1 continuity needs to be enforced at their interface [32,18,30,33].

As an application of IGA shell analysis, this work focuses on the reverse engineering of thin components manufactured through a deep drawing process and the simulation of their dynamic behaviour. In particular, a B-spline model is reconstructed starting from laser scan measurements on the surface of the components and, following the isogeometric framework, it is used directly for the numerical analysis.

Coordinate measuring machines (CMMs) are nowadays useful tools for reverse engineering and quality inspection applications [34–36] and, within this family, laser CMMs can produce large amounts of point data in a short time period without entering in contact with the scanned surface and are widely adopted for many applications in industry. However, the point cloud itself is most of the time not sufficient for an accurate description of the component or for the purpose of numerical analysis and, therefore, a surface has to be reconstructed through a reverse engineering process [37,35,38]. In this context, point cloud fitting with parametric curves and surfaces, such as B-splines and NURBS, has been employed starting much earlier than the introduction of IGA [3,39] because of the wide use of these representations in the CAD industry and also, from a mathematical point of view, because of the high-order and tailorable smoothness of splines functions.

Combining the B-spline reconstruction with IGA shell analysis can therefore be a useful tool for the numerical modelling and simulation of a component and this work considers the specific case of deep drawn components. The outline of the paper is the following: B-spline basis functions and surfaces are introduced in Section 2; then the surface reconstruction procedure is discussed in Section 3.1 and the method used in this work is presented; a brief discussion of IGA shell analysis is given in Section 4; two practical examples are studied in Section 5, where the IGA results are compared with experiments; concluding remarks follow in Section 6.

2. Fundamentals of B-spline curves and surfaces

In one dimension, a B-spline function of polynomial order p is defined by a knot vector $\Xi = \{\xi_1 \leqslant, \ldots, \leqslant \xi_{n+p+1}\}$, with n the number of basis functions forming the B-spline. The i^{th} $(i = 1, \ldots, n)$ B-spline basis function $N_i^p(\xi)$ is derived from the knot-vector using the Cox-De Boor recursive formula [3]:

$$N_i^0(\xi) = \begin{cases} 1 & \text{if} \quad \xi_i \leqslant \xi < \xi_{i+1}, \\ 0 & \text{otherwise}, \end{cases}$$
$$N_i^p(\xi) = \frac{\xi - \xi_i}{\xi_{i+p} - \xi_i} N_i^{p-1}(\xi) + \frac{\xi_{i+p+1} - \xi}{\xi_{i+p+1} - \xi_{i+1}} N_{i+1}^{p-1}(\xi), \quad \text{for } p \geqslant 1. \tag{1}$$

Given two univariate B-spline basis functions $N_i^p(\xi)$ $(i = 1, \ldots, n)$ and $M_j^q(\eta)$ $(j = 1, \ldots, m)$, of order p and q respectively, and associated to the two knot vectors $\Xi = \{\xi_1 \leqslant, \ldots, \leqslant \xi_{n+p+1}\}$ and $\mathcal{H} = \{\eta_1 \leqslant, \ldots, \leqslant \eta_{m+q+1}\}$, a bivariate B-spline basis function is defined as

$$R_{i,j}^{p,q}(\xi, \eta) = N_i^p(\xi) M_j^q(\eta) \tag{2}$$

and B-spline surfaces $\boldsymbol{S}(\xi, \eta)$ are defined by

$$\boldsymbol{S}(\xi, \eta) = \sum_{i=1}^{n} \sum_{j=1}^{m} R_{i,j}^{p,q}(\xi, \eta) \mathbf{B}_{i,j}, \tag{3}$$

where $\mathbf{B}_{i,j}$ are the control points. Or, with $k = i + (j-1)m$ a single counter replacing i and j,

$$\boldsymbol{S}(\xi, \eta) = \sum_{k=1}^{N} R_k(\xi, \eta) \mathbf{B}_k. \tag{4}$$

3. B-spline surface fitting

3.1. Point cloud fitting

In the context of B-spline and NURBS surface fitting, least squares methods have been the most used approach, starting from the first works dated back to the seventies [40,41]. The main idea of these methods is to consider a reference surface and to update the position of its control points such that the error between the surface and the point cloud is minimized.

Given a cloud to be fitted composed of M points denoted by $\mathbf{P}_l$, this consists in imposing M equations

$$\mathbf{P}_l = \sum_{k=1}^{N} R_k(\xi_l, \eta_l) \mathbf{B}_k, \quad l = 1, \ldots, M \tag{5}$$

or in matrix form

$$\mathbf{RB} = \mathbf{P}, \tag{6}$$

where $\mathbf{P}_l = (x_l, y_l, z_l)$ and ξ_l and η_l are the parametric coordinates associated to $\mathbf{P}_l$, whose computation is discussed in the following.

Since in general $M \gg N$, Eq. (6) corresponds to an overdetermined system of equations, which is solved in a least square sense [42]. Such a choice is also motivated by statistical reasoning, as it effectively reduces the influence of random errors in measurements. In practice, this corresponds to obtaining a square system by pre-multiplying both sides by $\mathbf{R}^T$:

$$\mathbf{R}^T \mathbf{RB} = \mathbf{R}^T \mathbf{P}. \tag{7}$$

The solution of this system minimizes the functional

$$f_s = \|\mathbf{RB} - \mathbf{P}\|_2^2, \tag{8}$$

corresponding to the sum of the squares of the distance from the points to the surface. Other approaches, based on different norms such as the L^1 and the L^∞ are also possible [43,44].

In the numerical practice, the direct solution of system (7) results to be accurate only in some limited cases, such as when a regular distributed set of points is considered. However, the point clouds coming from 3D scanners are composed by randomly oriented points, and regions with a lower density of points or holes can also be present, which results in a poor conditioning and sometimes even in a rank deficiency of $\mathbf{R}^T\mathbf{R}$. As a consequence, the fitted surface can present an irregular and oscillatory behaviour.

Therefore, in order to improve the quality of the fitting, regularizing terms are often added to functional (8). In particular, a common practice in curve and surface fitting consists in fairing by means of energy terms [45–49]. In fact, a spline surface can be seen as a mathematical representation of a membrane or an elastic plate and when it oscillates, the corresponding membrane and bending energy becomes higher. These energy terms are in general a nonlinear function of the surface derivatives and, by following a common practice of surface fairing, the corresponding linearised terms are considered in this work:

$$E_m(\boldsymbol{S}) = \iint \|\boldsymbol{S}_\xi\|^2 + \|\boldsymbol{S}_\eta\|^2 d\xi d\eta, \tag{9}$$

$$E_b(\boldsymbol{S}) = \iint \|\boldsymbol{S}_{\xi\xi}\|^2 + 2\|\boldsymbol{S}_{\xi\eta}\|^2 + \|\boldsymbol{S}_{\eta\eta}\|^2 d\xi d\eta, \tag{10}$$