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Rheological-dynamical analogy for analysis of vibrations and low cycle fatigue in internally damped inelastic frame structures

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ABSTRACT

This is a study of viscoelastoplastic (VEP) vibrations and their use for the analysis of low cycle fatigue in internally damped inelastic frame structures (IDIFSs). The background of this inelastic theory is presented in the framework of a mathematical-physical analogy between the rheological model and a dynamical model with viscous damping. The rheological-dynamical analogy (RDA) is a type of inelastic analysis, which transforms one category of material non-linear problems to simpler linear dynamical problems using modal analysis. The aim of this paper is to define internal damping based on both the dynamic modulus and modal damping ratios. The idea underlying these approaches is that fatigue damage appears if internal damping is unevenly distributed over the elements of a structure. The residual force method, which requires the use of the finite element method (FEM), is used for the location of damage and derivation of the fatigue damage vector. Finally, the effective force vector is derived from damage mechanics. An analysis of damaged IDIFSs made of reinforced concrete is carried out. It is shown that the RDA, which correlates with the main mechanical properties of the material measured, can improve the prediction of fatigue damage caused by low cycle fatigue.

1. Introduction

In practice, engineering structures are usually complex, and their dynamic analysis is traditionally performed using the conventional FEM. Dynamic analysis in structural engineering falls into two different classes, one involving low frequency loading, and the other high frequency loading. Low frequency problems are categorized as structural dynamics problems, where the frequency content of the dynamic load is of the order of a few hundred hertz and the designer will be mostly interested in its long-term (or steady-state) effects on the structure. This paper is concerned with a new proposal regarding the analysis of low cycle fatigue in IDIFSs.

Finite element solutions in dynamics are obtained by employing two different methods [1,2]; the modal method and time marching schemes. In modal analysis responses of individual modes are superimposed to determine the total response. Traditionally, energy dissipation in a structure is represented as an idealized viscous damping force, i.e. a force directly proportional to the velocity of the corresponding dynamic system. In this case, the structure mass and stiffness matrices remain constant during the analysis and satisfy the well-known orthogonality conditions. If the damping matrix also satisfies the criterion of orthogonality, the equations of motion for a discretized multi-degree-of-freedom (MDOF) structure can be decoupled into i independent equations, one for each normal mode of the structure. This is equivalent to assuming that the normal modes of a damped system are identical to those of an undamped system. It is well known that the damping matrix is not diagonal for all real structures; nevertheless, in order to uncouple the modal equations, it is necessary to assume that there is no coupling between the modes. Therefore, it is assumed that the damping matrix is diagonal, with the VEP modal damping terms $c_i = 2\xi_i m_i \omega_i$. Ratio ξ_i is defined as the ratio of damping in mode i to the critical damping in mode i. In most cases modal damping ratios ξ_i are used in computer modeling to approximate unknown energy dissipation within a structure.

Nowadays, there is a growing interest in developing a theory which would enable the prediction of fatigue damage in structures. Cumulative fatigue damage analysis plays a key role in the life prediction for components and structures with field load histories [3]. However, investigation in this paper shows that internal damping that is unevenly distributed over the elements of an IDIFS causes deterioration of the material named fatigue damage. This analysis is based on the RDA. The RDA inelastic theory has been developed to describe the dynamic response of structures using both the

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dynamic modulus and modal damping ratios [4,5]. The dynamic modulus is obtained based on a concept of a complex modulus of VEP materials, whereas the modal damping ratios are obtained by observing critically damped dynamic systems in the steady-state response. It has been proved that the dynamic modulus is equal to the tangent modulus at selected moments in time in some plastic materials [6]. Also, internal damping is a significant factor, considered as a damage variable in low cycle fatigue. The eigenvalues of a structure must first be solved for the dynamic system relieved of external masses, which is required to critically damp it [4]. This is necessary in order to calculate the modal damping ratios for systems composed of consistent or lumped external masses. A system composed of external masses has its own eigenvalues. Also, the RDA is an analytical method whereby resonant frequencies may simply be calculated using zero modal damping ratios.

The structural damage detection method, which uses the modal parameters of dynamic systems, has been in use for a long time under two approaches: the first approach relates variation in the strain energy of a structure to changes in its structural frequencies [7], and the second approach relates changes in the stiffness and mass properties of a structure to changes in its structural frequencies and mode shapes. The second approach, named the residual force method [8], is the more versatile method. In this paper, a new methodology is presented in the framework of the residual force method using the RDA. The proposed algorithm combines the RDA and continuum damage mechanics to derive the fatigue damage vector of structures, as well as the effective force vector. Damage is observed on the macro scale. It is assumed at this scale that materials consist of continuously distributed material points, and that material density as well as other relevant physical properties of materials can be determined. When cyclic load is applied to a material, micro cavities may be induced as a result of irreversible plastic deformation. These cavities may grow to form cracks and give rise to the final failure of the material. The deterioration of the material in this process is called fatigue damage [9]. The damage caused by low cycle fatigue was verified in a typical IDIFS made of reinforced concrete using three damaged structures. The aim was to demonstrate the validity and applicability of the RDA inelastic theory. Numerical examples are provided to show theoretical considerations, confirming that the RDA improves the prediction of fatigue damage in IDIFSs.

2. Rheological-dynamical analysis of vibrations

2.1. RDA – a short overview

Material micro cracking is accompanied by the loading of a specimen, leading to its damage and failure. Consider the case of the VEP strain of a rod presented in Fig. 1a). In material investigations, both stress $\sigma(t)$ and inelastic strain $\varepsilon^*(t) = \varepsilon_{ve}(t) + \varepsilon_{vp}(t)$ are functions of time. If the total VEP strain $\varepsilon(t) = \varepsilon_{el} + \varepsilon^*(t)$ is presented as a sum of elastic (instantaneous), viscoelastic (VE) and viscoplastic (VP) components, each isochronous stress-strain diagram of a thin long symmetrical rod (e.g., with a square or circular cross section A_0) can accurately be approximated by the VEP rheological model H-K-(StV|N), consisting of five elements. The rheological model is shown in Fig. 1b) using the following symbols: N for the Newtonian dashpot, StV for Saint-Venant's body, H for the Hookean spring, "|" for a parallel connection and "—" for a connection in a series.

Since the Hookean spring, Kelvin's body (K = H|N) and VP body (StV|N) are connected in a series, stresses $\sigma(t)$ in all the models are equal. Based on the VEP rheological model, Milašinović [10] derived a governing differential equation,

$$\ddot{\varepsilon}(t) + \dot{\varepsilon}(t)\left(\frac{E_K}{\lambda_K} + \frac{H'}{\lambda_N}\right) + \varepsilon(t)\frac{E_K H'}{\lambda_K\lambda_N} = \frac{\ddot{\sigma}(t)}{E_H} + \dot{\sigma}(t)\left(\frac{E_K}{\lambda_K E_H} + \frac{H'}{\lambda_N E_H} + \frac{1}{\lambda_K} + \frac{1}{\lambda_N}\right) + \sigma(t)\left(\frac{E_K}{\lambda_K\lambda_N} + \frac{H'}{\lambda_K\lambda_N} + \frac{E_K H'}{\lambda_K\lambda_N E_H}\right) - \sigma_Y\frac{E_K}{\lambda_K\lambda_N}, \tag{1}$$

where E_H is the elastic modulus, σ_Y the uniaxial yield stress and $Y = \sigma_Y + H'\varepsilon_{vp}(t)$ the VEP yield condition. The four properties at fixed step times are: extensional VE viscosity λ_K, extensional VP viscosity λ_N, VE modulus E_K and VP modulus H'. However, these constants cannot easily be determined in physical experiments, especially Trouton's viscosities λ_K and λ_N. The corresponding homogeneous equation of the total VEP strain is as follows,

$$\ddot{\varepsilon}(t)\lambda_K\lambda_N + \dot{\varepsilon}(t)(E_K\lambda_N + H'\lambda_K) + \varepsilon(t)E_K H' = 0. \tag{2}$$

On the other hand, a mechanical longitudinal disturbance (strain) propagates in an elastic medium at the finite initial phase velocity $v_0 = (E_H/\rho)^{1/2}$, where ρ is the density of the medium. The vibration at an arbitrary point M of the rod lags in the phase behind that at the source of the wave. If l_0 is the initial distance between the two ends of the rod, the time required for a wave to travel from one to the other end of it is $t\text{-}t_0 = T^D = l_0/v_0$. The natural angular frequency ω of the discrete dynamical model, which represents the undamped free longitudinal vibration of a rod, is given by

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{E_H A_0}{l_0}\frac{1}{\rho A_0 l_0}} = \frac{v_0}{l_0} = \frac{1}{T^D} \Rightarrow T^D = \frac{l_0}{v_0} = \frac{1}{\omega}. \tag{3}$$

Here, m is the mass of the rod and k its axial stiffness, as shown in Fig. 1c.

Bearing in mind Eq. (2), an expression similar to Eq. (3) can be formulated, setting the rheological model of the rod into the state of critical viscous damping ($c = c_{cr}$),

$$\omega = \sqrt{\frac{E_K H'}{\lambda_K\lambda_N}} = \sqrt{\frac{1}{T_K T^*}} = \frac{1}{T^D}, \tag{4}$$

where $E_K/\lambda_K = H'/\lambda_N$, $\lambda_K = E_K T_K$, $\lambda_N = H'T^*$ and $T_K = T^* = T^D$.

Therefore,

$$\sqrt{\frac{E_H}{\rho}}\frac{1}{l_0} = \sqrt{\frac{E_K H'}{\lambda_K\lambda_N}} \Rightarrow \lambda_K\lambda_N = \frac{E_K H'\gamma l_0^2}{E_H g} \Rightarrow \frac{\lambda_K\lambda_N}{\gamma} = \frac{E_K H' A_0 l_0^2\rho}{E_H\gamma A_0}, \tag{5}$$

where γ is the specific gravity. Thus,

$$m = \frac{\lambda_K\lambda_N}{\gamma} = k(T^D)^2, \quad k = \frac{E_K H'}{\gamma}, \quad c_{cr} = 2\sqrt{km} = 2kT^D. \tag{6}$$

Consequently, the propagation of longitudinal elastic waves forms the physical basis for the analogy between two different physical phenomena, the rheological and the dynamical. Then, Eq. (2) may be expressed as follows,

$$\ddot{\varepsilon}(t)m + \dot{\varepsilon}(t)c_{cr} + \varepsilon(t)k = 0. \tag{7}$$

Therefore, a very complicated non-linear problem in the VEP range of strains may be solved as a simple linear dynamic one. Although the RDA is derived in order to solve dynamic problems [4,5], it can be used in the analysis of quasi-static loading ($\delta \to 0$) considering the corresponding limit values of derived analytical expressions. Hence, each quasi-static stress-strain curve of a specimen (rod, column, beam, etc.) can be obtained using the RDA modulus function [6].