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A stochastic boundary element method for piezoelectric problems

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A B S T R A C T

In this paper, the stochastic boundary element method of piezoelectric problems with randomness of material parameters and randomness of applied loads is proposed. By using the method of first-order Taylor expansion, each random quantity is written into the sum of the mean and deviation. The boundary integral equations corresponding to the means and deviations of the displacements and electric potential are derived, respectively. It is demonstrated that the randomness of material parameters can be transformed into the equivalent random body forces and equivalent random charge density, so that the fundamental solutions of deterministic piezoelectric problems can be used in boundary integral equations of the means or deviations due to the similarity between the governing equations with randomness and those of deterministic piezoelectric problems. Finally, several numerical examples are performed to verify the validity of the proposed stochastic boundary element method.

1. Introduction

The piezoelectric components have been widely used in engineering because of its unique positive and inverse piezoelectric effect. However, in practical engineering, it is always unavoidable to introduce some uncertainties in the material properties and the applied loads during the manufacturing and loading processes. In some cases such uncertainties may be significant and should not be ignored. These uncertainties are usually spatially distributed and correlated over the whole structures and should be modeled as random fields. In this case, the relationship between deformation and external load should be described by stochastic partial differential equations with random coefficients. Therefore, in order to analyze the strength and reliability of piezoelectric components with such uncertainties more effectively, stochastic finite element method (SFEM) and stochastic boundary element method (SBEM) were developed to analyze such structures combined with methods of probability statistics. Vehoosol and Gutierrez [1] indicated that with the size decrease of micro electromechanical systems (MEMS) made of piezoelectric ceramics, controllability of production processes becomes increasingly difficult, and then the structural properties of MEMS are subject to the large uncertainties. By means of the SFEM, it is demonstrated that these uncertainties can have a significant influence on both the performance and reliability of the piezoelectric components. Srivastava et al. [2] studied the influence of randomness of material properties on the performance of piezoelectric fan by using the Monte Carlo Simulation. The effect of these uncertainties must be taken into account in the design and modification process of the piezoelectric fan. The results of this study will enhance confidence in the design process and will eventually pave the way for improved micro-electronic cooling systems.

As compared with SFEM, SBEM has emerged as a powerful tool for structural analysis with the uncertainties due to its unique advantages, for example, the discretization of the boundary that leads to less computing work and the use of fundamental solutions for the infinite media that leads to a higher accuracy. And it is more efficient for the fracture and fatigue problems for piezoelectric ceramics due to their intrinsic brittleness [3].

The earliest research work on SBEM, to the best of authors' knowledge, was started by Burczytiski [4], and he solved the stochastic boundary value problems of elasticity. Later, the SBEM was extended to different fields. For potential problems, Nakagiri et al. [5] used perturbation technique to derive SBEM, taking account of only the random boundary shape. Kaljevic and Saigal [6] studied two-dimensional steady state potential flow, taking account of random geometric configuration and random material parameter. For heat conduction problems, Drewniak [7] studied heat conduction problems with random conductivity and random heat transfer coefficient. For elastic problems, Ren et al. [8] presented the SBEM for static problems by using the first-order Taylor expansion method. Burczynski and Skrzypczyk [9] proposed the SBEM to static and dynamic problems with random mixed boundary conditions, random material parameters and stochastic geometry

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boundary including external and internal boundaries. Wang et al. [10,11] extended the SBEM to the sufficient and necessary boundary element method for both potential problem and plane problem of elasticity. Kaminski [12] extended the SBEM to composite materials with stochastic interface defects. There were also a lot of works focusing on groundwater flow problems [13–16]. In recent years, Su and coworkers [17,18] presented stochastic spline fictitious boundary element method and analyzed the reliability of plane elasticity problem and plate bending problem. They also discussed the random vibration of plane elastic problems with both structural and loading uncertainties [19]. So far, there has been no report on applying SBEM to the piezoelectric problem.

The main purpose of this paper is to propose the stochastic boundary element method to analyze piezoelectric components with random material parameters and random applied loads. The boundary integral equations corresponding to the means and deviations of the displacements and electric potential are derived. Furthermore, the covariance matrices of the unknown boundary displacements and electric potentials as well as surface tractions and surface charges can be obtained. Finally several numerical examples of piezoelectric plane problems are presented to verify the efficiency of the proposed SBEM.

2. Basic equations of piezoelectricity and BEM formulations

The governing equations for piezoelectricity can be summarized as follows:

$$\left.\begin{aligned}\sigma_{ij,j}+f_i=0\\ D_{i,i}-q=0\end{aligned}\right\}, \tag{1}$$

where σ_{ij} are the components of elastic stress, D_i are the three electric displacement components; f_i are the three components of body force per unit volume and q is the charge density.

The constitutive relations between the stresses and electric displacements with the displacements and electric potential for the piezoelectricity are [20]:

$$\begin{aligned}\sigma_{ij}&=C_{ijkl}u_{k,l}+e_{kij}\phi_{,k}\\ D_i&=e_{ikl}u_{k,l}-\varepsilon_{ik}\phi_{,k},\end{aligned} \tag{2}$$

where u_i are the three elastic displacement components and ϕ is the electric potential; c_{ijkl} , e_{ijk} and ε_{ij} are the elastic, piezoelectric and dielectric material constants, respectively. Moreover, partial differentiation with respect to a space variable is denoted with a comma.

The boundary conditions are given by

$$\left.\begin{aligned}\sigma_{ij}n_j=\tilde{t}_i \; on\, \Gamma_t\\ u_i=\tilde{u}_i\; on\,\Gamma_u\end{aligned}\right\}\ \left.\begin{aligned}D_in_i=-\tilde{\omega}\; on\,\Gamma_\omega\\ \phi=\tilde{\phi}\; on\,\Gamma_\phi\end{aligned}\right\}, \tag{3}$$

where t_i are the components of the surface traction, ω is the surface charge, n_i is the unit outward normal vector, and "∼" indicates the prescribed value. Note that $\Gamma_t+\Gamma_u=\Gamma_\omega+\Gamma_\phi=\Gamma$.

If material parameters can be modeled as the functions of the spatial coordinates, the above equations can result in equilibrium equations in terms of the displacements and electric potential for the transversely isotropic piezoelectricity (the x-y plane is isotropic plane, and the z-axis is the polarization direction of piezoelectric materials) in the matrix form as follows:

$$[\boldsymbol{D}]\begin{Bmatrix}u_1\\u_2\\u_3\\\phi\end{Bmatrix}+[\boldsymbol{D}_1]\begin{Bmatrix}u_1\\u_2\\u_3\\\phi\end{Bmatrix}+\begin{Bmatrix}f_1\\f_2\\f_3\\-q\end{Bmatrix}=0, \tag{4}$$

where $[\boldsymbol{D}]$ and $[\boldsymbol{D}_1]$ are the differential operator matrices given by:

$$[\boldsymbol{D}]=\begin{bmatrix} c_{11}\frac{\partial^2}{\partial x^2}+c_{66}\frac{\partial^2}{\partial y^2}+c_{44}\frac{\partial^2}{\partial z^2}, & (c_{12}+c_{66})\frac{\partial^2}{\partial x\partial y}, & (c_{13}+c_{44})\frac{\partial^2}{\partial x\partial z}, & (e_{15}+e_{31})\frac{\partial^2}{\partial x\partial z}\\ (c_{12}+c_{66})\frac{\partial^2}{\partial x\partial y}, & c_{66}\frac{\partial^2}{\partial x^2}+c_{11}\frac{\partial^2}{\partial y^2}+c_{44}\frac{\partial^2}{\partial z^2}, & (c_{13}+c_{44})\frac{\partial^2}{\partial y\partial z}, & (e_{15}+e_{31})\frac{\partial^2}{\partial y\partial z}\\ (c_{13}+c_{44})\frac{\partial^2}{\partial x\partial z}, & (c_{13}+c_{44})\frac{\partial^2}{\partial y\partial z}, & c_{44}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)+c_{33}\frac{\partial^2}{\partial z^2}, & e_{15}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)+e_{33}\frac{\partial^2}{\partial z^2}\\ (e_{15}+e_{31})\frac{\partial^2}{\partial x\partial z}, & (e_{15}+e_{31})\frac{\partial^2}{\partial y\partial z}, & e_{15}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)+e_{33}\frac{\partial^2}{\partial z^2}, & -[\varepsilon_{11}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)+\varepsilon_{33}\frac{\partial^2}{\partial z^2}] \end{bmatrix}, \tag{5}$$

and

$$[\boldsymbol{D}_1]=\begin{bmatrix} c_{11,x}\frac{\partial}{\partial x}+c_{66,y}\frac{\partial}{\partial y}+c_{44,z}\frac{\partial}{\partial z}, & c_{66,y}\frac{\partial}{\partial x}+c_{12,x}\frac{\partial}{\partial y}, & c_{44,z}\frac{\partial}{\partial x}+c_{13,x}\frac{\partial}{\partial z}, & e_{15,z}\frac{\partial}{\partial x}+e_{31,x}\frac{\partial}{\partial z}\\ c_{12,y}\frac{\partial}{\partial x}+c_{66,x}\frac{\partial}{\partial y}, & c_{66,x}\frac{\partial}{\partial x}+c_{11,y}\frac{\partial}{\partial y}+c_{44,z}\frac{\partial}{\partial z}, & c_{44,z}\frac{\partial}{\partial y}+c_{13,y}\frac{\partial}{\partial z}, & e_{15,z}\frac{\partial}{\partial y}+e_{31,y}\frac{\partial}{\partial z}\\ c_{13,z}\frac{\partial}{\partial x}+c_{44,x}\frac{\partial}{\partial z}, & c_{13,z}\frac{\partial}{\partial y}+c_{44,y}\frac{\partial}{\partial z}, & c_{44,x}\frac{\partial}{\partial x}+c_{44,y}\frac{\partial}{\partial y}+c_{33,z}\frac{\partial}{\partial z}, & e_{15,x}\frac{\partial}{\partial x}+e_{15,y}\frac{\partial}{\partial y}+e_{33,z}\frac{\partial}{\partial z}\\ e_{31,z}\frac{\partial}{\partial x}+e_{15,x}\frac{\partial}{\partial z}, & e_{31,z}\frac{\partial}{\partial y}+e_{15,y}\frac{\partial}{\partial z}, & e_{15,x}\frac{\partial}{\partial x}+e_{15,y}\frac{\partial}{\partial y}+e_{33,z}\frac{\partial}{\partial z}, & -\left(\varepsilon_{11,x}\frac{\partial}{\partial x}+\varepsilon_{11,y}\frac{\partial}{\partial y}+\varepsilon_{33,z}\frac{\partial}{\partial z}\right) \end{bmatrix}. \tag{6}$$

If the material parameters are deterministic constants, then Eq. (4) can be simplified to

$$[\boldsymbol{D}]\begin{Bmatrix}u_1\\u_2\\u_3\\\phi\end{Bmatrix}+\begin{Bmatrix}f_1\\f_2\\f_3\\-q\end{Bmatrix}=0. \tag{7}$$

The boundary integral equations for the above problem have been obtained in Ref. [21], which takes the following form if we use the extended notations u_J, t_J and f_J [22] representing the generalized displacements, surface tractions and body forces, respectively

$$\begin{aligned}&c_{IJ}(\xi)u_J(\xi)+\int_\Gamma t^*_{IJ}(\xi,x)u_J(x)d\Gamma(x)\\&=\int_\Gamma u^*_{IJ}(\xi,x)t_J(x)d\Gamma(x)+\int_\Omega u^*_{IJ}(\xi,x)f_J(x)d\Omega(x)\\&(I,\ J=1,\ 2,\ 3,\ 4),\end{aligned} \tag{8}$$

where

$$\begin{cases}u_J=u_j, & (J=j=1,\ 2,\ 3)\\ u_J=-\phi, & (J=4)\end{cases},\ \begin{cases}t_J=t_j, & (J=j=1,\ 2,\ 3)\\ t_J=-\omega, & (J=4)\end{cases},$$

$$\begin{cases}f_J=f_j, & (J=j=1,\ 2,\ 3)\\ f_J=-q, & (J=4)\end{cases}$$

$$u^*_{IJ}=\begin{cases}u^*_{Ij},(I=1,2,3,4\ \ J=j=1,2,3)\\ \phi^*_I,(I=1,2,3,4\ \ J=4)\end{cases},$$